MT240: Complex Variables and Transforms

Worksheet 7

Fri, Mar 15 *Spring 2019*

Roll# Student 1: Roll# Evaluator 1:

Roll# Student 2: Roll# Evaluator 2:

Problem 1 [15 minutes]

For the Möbius transformations $f(z) = \dfrac{5z-3}{z+i}$, answer the following. For each part, draw the pre-image in z-plane and its image in w-plane. Use the same z- and w-planes for parts (i)-(iv). Then make separate zs and w-planes and use these for parts (vi)-(ix).

(i) Evaluate $f(0)$

(ii) Evaluate $\lim_{z\to\infty} f(z)$

(iii) Find the value of z where $f(z) = 0$.

(iv) Find the value of z where $f(z) \to \infty$

(v) Find the inverse function $f^{-1}(w)$. Let $g(z) = f^{-1}(z)$.

(vi) Evaluate $g(0)$

(vii) Evaluate $\lim_{z\to\infty} g(z)$

(viii) Find the value of z where $g(z) = 0$

(ix) Find the value z where $g(z) \to \infty$

(x) Compare parts (i)-(iv) with parts (vi)-(ix). What can you conclude about your answers?

Problem 2 [5 minutes]

Find a Möbius transformation which maps the points $z_1 = 0$, $z_2 = -i$ and $z_3 = i$ to 0, 1 and ∞ respectively.

Problem 3 [15 minutes]

Draw the map of the circle $|z-1| = 1$ under the transformation

$$f(z) = \frac{6z+6}{2z+1}$$

Problem 4 [25 minutes]

Evaluate the integral

$$\int_\gamma (2z + iz^2)dz$$

over each of the following curves using curve parametrization.

(a) γ : from $z = 0$ to $z = 2-2i$ along the line $y = -x$

(b) γ : from $z = -1$ to $z = 1$ in the upper half of unit circle centered at origin

(c) γ : from $z = -1$ to $z = 1$ in the lower half of unit circle centered at origin

(d) γ : from $z = 1$ back to $z = 1$ counterclockwise in a circle of radius 2 centered at $1 + 2i$.

Problem 5 [15 minutes]

Solve Problem 4 using the fundamental theorem of calculus and the concept of *primitive* and path independence in the complex plane.