MT240: Complex Variables and Transforms

Worksheet 5

Fri, Mar 1 *Spring 2019*

Roll# Student 1: Roll# Evaluator 1:

Roll# Student 2: Roll# Evaluator 2:

Problem 1

For each of the following functions,

(a) $f(z) = z^5$ *

(b) $f(z) = e^{-x-iy}$

(c) $f(z) = 3x + y + i(3y - x)$

(d) $f(z) = e^{-y} \sin x - ie^{-y} \cos x$

(i) Show that the function is *entire*, i.e. analytic everywhere in the complex plane.
[Hint: Theorem 2.3 in the Notes. Consider in the polar form the functions with '*' symbol.]

(ii) Find its derivative $f'(z)$ using partial derivatives. [Hint: Theorem 2.2 in the Notes]

(iii) Now find its derivative $f'(z)$ using differentiation in z and compare your answer with (ii).

(iv) Because the function is entire, its component functions $u(x, y)$ and $v(x, y)$ must be *harmonic* everywhere in the complex plane. Verify that the functions $u(x, y)$ and $v(x, y)$ are harmonic $\forall z \in \mathbb{C}$. [Hint: Theorem 2.5 in the Notes]

(v) If the function is written in x and y, convert it in terms of z, without making use of $\overline{z}$.

Problem 2

For each of the following functions, determine the values of z for which $f'(z)$ exists, i.e. the function is analytic, and find $f'(z)$ for these values.

(a) $f(z) = x^2 + iy^2$

(b) $f(z) = (x + iy)y$

Problem 3

Are the following functions harmonic? If your answer is yes, find a corresponding analytic function $f(z) = u(x, y) + iv(x, y)$.

(a) $u(x, y) = y^3 - 3x^2y$

(b) $v(x, y) = 3y - x$

(c) $v(x, y) = e^x \sin 2y$

(d) $u(x, y) = \dfrac{y}{x^2 + y^2}$

(e) $u(x, y) = e^{-y} \sin x$

(f) $v(x, y) = x^2 - y^2 + 2y$