MT240: Complex Variables and Transforms

Worksheet 4

Fri, Feb 22 *Spring 2019*

Roll# Student 1: Roll# Evaluator 1:

Roll# Student 2: Roll# Evaluator 2:

Problem 1

Under the function $f(z) = e^z$, sketch the image of each of the following sets.

(a) $\{z \in \mathbb{C} : \text{Re } z = 2\}$ (b) $\left\{z \in \mathbb{C} : \text{Im } z = \frac{\pi}{4}\right\}$ (c) $\left\{z \in \mathbb{C} : \frac{\pi}{4} < \text{Im } z < \frac{3\pi}{4}\right\}$

Problem 2

Evaluate the following limits. Also specify whether the function is continuous or not at the point where you evaluated the limit.

(a) $\displaystyle\lim_{z\to -1+i} \frac{z^2+2z+2}{z+1}$ (b) $\displaystyle\lim_{z\to i} \frac{z^2+1}{z(z-i)}$ (c) $\displaystyle\lim_{z\to 0} \frac{z^2}{z}$

Problem 3

For the function $f(z) = \dfrac{z^2}{|z|^2}$, evaluate the following limits in the complex plane.

(a) $\displaystyle\lim_{z\to 0}$ along the positive x-axis

(b) $\displaystyle\lim_{z\to 0}$ along the positive y-axis

(c) $\displaystyle\lim_{z\to 0}$ along the line $y = x$

(d) $\displaystyle\lim_{z\to 0}$ along the line $y = 2x$

Problem 4

Using first principles, find the derivative of the following complex functions at a point $z = z_0$.

(a) $f(z) = 2z + 1$ (b) $f(z) = 2z^2 - z$

Problem 5

For each of the following functions,

(a) $f(z) = z^2$

(b) $f(z) = e^{-z}$

(c) $f(z) = 3x + y + i(3y - x)$

(d) $f(z) = e^{-y}\sin x - ie^{-y}\cos x$

(i) Show that the function is 'entire', i.e. analytic everywhere in the complex plane.
[Hint: Theorem 2.3 in the Notes]

(ii) Find its derivative $f'(z)$ using partial derivatives.
[Hint: Theorem 2.2 in the Notes]

(iii) If the function is written in x and y, convert it in terms of z.

(iv) Find its derivative $f'(z)$ using differentiation in z and compare your answer with (ii).