



## Worksheet 3

Fri, Feb 15

Spring 2019

Roll# Student 1:

Roll# Evaluator 1:

Roll# Student 2:

Roll# Evaluator 2:

**Problem 1**

- (a)  $\{z \in \mathbb{C} : |z| < 1 \vee |z - 4i| < 3\}$                       (b)  $\{z \in \mathbb{C} : -\pi < \text{Arg } z < \pi\}$

For each of the above sets, answer the following.

- (i) Sketch the set in the complex plane.
- (ii) Describe the boundary of the set using the set-builder notation.
- (iii) Describe the interior of the set using the set-builder notation.
- (iv) Is the set open or closed? Explain.
- (v) If the set is open, is it connected? Explain.

**Problem 2**

For each of the following functions, describe its domain in  $\mathbb{C}$ , i.e. set of values of  $z$  for which the function is defined.

- (a)  $f(z) = z + 1$     (e)  $f(z) = \text{Arg } \frac{1}{z}$   
 (b)  $f(z) = \bar{z}$     (f)  $f(z) = \frac{z}{z + \bar{z}}$   
 (c)  $f(z) = \frac{1}{z - 1 - 6i}$     (g)  $f(z) = \frac{1}{1 - |z|^2}$   
 (d)  $f(z) = \frac{1}{z^2 + 2z + 2}$

**Problem 3**

Convert the following functions in the form  $f(z) = u(x, y) + iv(x, y)$ .

- (a)  $f(z) = z + i$     (d)  $f(z) = \frac{1}{z}$   
 (b)  $f(z) = 5iz$     (e)  $f(z) = -iz^2$   
 (c)  $f(z) = z + i\bar{z}$     (f)  $f(z) = e^{2iz}$

## Problem 4

Sketch the region onto which the set  $\left\{z : |z| < 2 \text{ and } 0 \leq \text{Arg } z \leq \frac{\pi}{4}\right\}$  is mapped by each of the following functions. Specify the range in set notation in each case.

(a)  $f(z) = z^2$

(b)  $f(z) = z^{-1}$

(c)  $f(z) = z^{\frac{1}{2}}$

## Problem 5

Sketch the image of these regions

(a)  $\{z : -1 < \text{Re } z < 2\}$

(b)  $\{z : -1 < \text{Re } z < 2 \wedge 0 < \text{Im } z < \frac{\pi}{2}\}$

under each of the following mapping function. Represent the range of the function in set notation in each case.

Tip: First map the boundary of domain to  $u$ - $v$ . Then choose an interior point of the domain, map it to  $u$ - $v$  plane and see on what side of the boundary does it get mapped in the  $u$ - $v$  plane.

(i)  $f(z) = z - 2$

(iii)  $f(z) = iz$

(ii)  $f(z) = 3z$

(iv)  $f(z) = e^{iz}$

## Problem 6

Sketch the image of each of the following functions defined on given domains and write down the range in set notation.

Tip: First map the boundary of domain to  $u$ - $v$ . Then choose an interior point of the domain, map it to  $u$ - $v$  plane and see on what side of the boundary does it get mapped in the  $u$ - $v$  plane.

(a)  $f(z) = z^2$ , defined on  $\{z : |z| < 4 \wedge |\text{Arg } z| < \frac{\pi}{12}\}$

(b)  $f(z) = \frac{1}{z}$ , defined on  $\{z : \frac{1}{4} < |z| < 4\}$

(c)  $f(z) = e^{-z}$  defined on  $\{z : -2 < \text{Re } z < 2 \wedge -\frac{\pi}{6} < \text{Im } z < \frac{\pi}{6}\}$