MT240: Complex Variables and Transforms

Worksheet 2

Fri, Feb 8

Spring 2019

Roll# Student 1: Roll# Evaluator 1:

Roll# Student 2: Roll# Evaluator 2:

Problem 1

(a) Evaluate the series $i^1, i^2, i^3, ..., i^{12}$ and observe the pattern that is emerging. Devise a general rule for evaluating i^n.

(b) Using your answer in (a), evaluate the following without using a calculator or a computer.

(i) i^{565} (ii) i^{6352} (iii) $i^{7853219}$ (iv) $(-i)^{1943278953}$

Problem 2

For each of the following complex polynomials,

(a) $z^4 + 2iz^3 + 2z^2$ (b) $z^3 - 27 + 27i$ (c) $z^4 + 1$ (d) $z^5 - 32i$

(i) Determine its number of roots using the Fundamental Theorem of Algebra.

(ii) Specify whether its complex roots, if any, are guaranteed to appear in conjugate pairs or not.

(iii) Find all the complex roots of the polynomial.

(iv) Plot the roots on the complex plane.

Problem 3

(a) $\{z \in \mathbb{C} : \text{Re } z > 4\}$

(b) $\{z \in \mathbb{C} : \text{Im} > -1\}$

(c) $\{z \in \mathbb{C} : 4 < \text{Re } z \leq 6\}$

(d) $\{z \in \mathbb{C} : -1 \leq \text{Im} < 0\}$

(e) $\{z \in \mathbb{C} : |z| = 1\}$

(f) $\{z \in \mathbb{C} : |z| < 1\}$

(g) $\{z \in \mathbb{C} : |z| > 1\}$

(h) $\{z \in \mathbb{C} : |z - i| < 2\}$

(i) $\{z \in \mathbb{C} : |z + i| < 2\}$

(j) $\{z \in \mathbb{C} : 1 < |z - i| < 2\}$

(k) $\{z \in \mathbb{C} : |z + 4i| \leq 2\}$

(l) $\{z \in \mathbb{C} : |z - 1 + i| < \sqrt{2}\}$

(m) $\{z \in \mathbb{C} : -\frac{\pi}{3} < \text{Arg } z < \frac{\pi}{3}\}$

(n) $\{z \in \mathbb{C} : \text{Re } \left(z^2\right) > 0\}$

For each of the above sets, answer the following.

(i) Sketch the set in the complex plane.

(ii) Is the set bounded? Explain.