MT240: Complex Variables and Transforms

Worksheet 11

Fri, Apr 19 *Spring 2019*

Roll# Student 1: Roll# Evaluator 1:

Roll# Student 2: Roll# Evaluator 2:

Problem 1 [20 minutes]

In this problem, or activity to be more precise, you will listen to square, sawtooth and triangular waves. If you are using headphones/earphones, first make sure the volume of your PC is not more than 20% otherwise your ears may hurt.

In MATLAB, open the file 'tones_fourier.m' uploaded on Classroom and modify lines 3-6 according to the following instructions.

1. Set fundamental frequency $f_o = 50$ Hz, i.e. set `freq_o = 50`.
2. You can choose the total number of harmonics N to be either 9 or 11, i.e. the total number of component frequencies you want to appear in the Fourier sum.
3. For square wave, use $a_n = \frac{1}{n}\sin(\frac{\pi n}{2}),\ b_n = 0$.
4. Press F5 to run the file and listen to the 1-second tones being played. You will first hear all the component tones of the square wave from f_o upto Nf_o and their graphs will be displayed simultaneously in the top panel. Then finally, you will listen to the approximate square wave and its graph will be displayed in the bottom panel.
5. Now repeat for different values of fundamental frequencies in the range 10 to 1000 Hz and describe your observations.

You can listen to other functions as well if you set a_n and b_n according to the following.

- Sawtooth wave 1: $a_n = 0,\ b_n = \frac{1}{n}(-1)^n$
- Sawtooth wave 2: $a_n = 0,\ b_n = \frac{2}{n}\left(1 + (-1)^n\right)$
- Triangle wave: $a_n = \frac{1}{n^2}\left(1 - (-1)^n\right),\ b_n = 0$

More fun: type the following in the command window and listen. Then try with 2*Fs or 3*Fs.

```
>>load handel.mat
>>sound(y,Fs)
```

Problem 2 [20 minutes]

Consider the following aperiodic function. This is known as 'pulse' function.

$$f(t) = \begin{cases} -1 & -\frac{a}{2} < t < 0 \\ 1 & 0 \le t < \frac{a}{2} \\ 0 & \text{otherwise} \end{cases}$$

(a) Sketch the graph of $f(t)$.

(b) Using absolute integrability test, show that its Fourier transform exists.

(c) Evaluate its Fourier transform $F(\omega)$.

(d) Sketch the graph of $F(\omega)$.

Problem 3 [20 minutes]

Consider the periodic function $f(t) = t$ for $t \in (-\pi, \pi)$ and $f(t - 2\pi k) = f(t)$ for $k \in \mathbb{Z}$ (this second condition just means that $f(t)$ has a time period of 2π).

(a) Sketch the graph of $f(t)$.

(b) Find its Fourier series coefficients c_n.

(c) Plot the power spectrum $|c_n|^2$ for $-5 \leq n \leq 5$.

(d) The following infinite sum

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots$$

seems difficult to evaluate. But it can be easily evaluated by applying Parseval's identity to $f(t)$ in this problem. Evaluate this inifnite sum, paying special attention to the limits of summation.