

# Homework 6

Due 8 am, Fri May 17

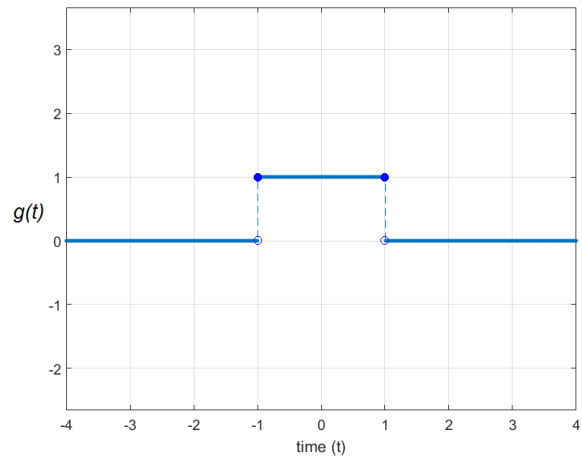
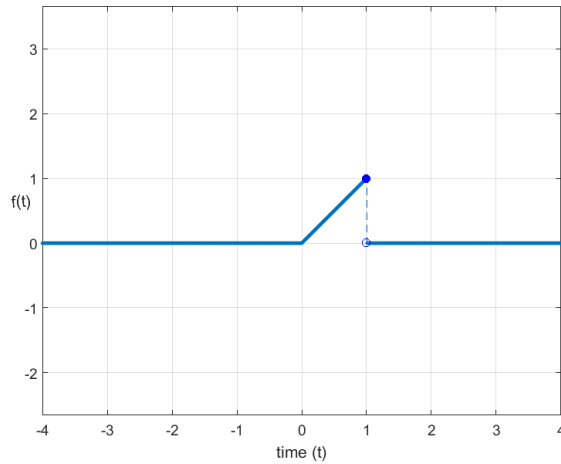
Spring 2019

## Tips to avoid plagiarism

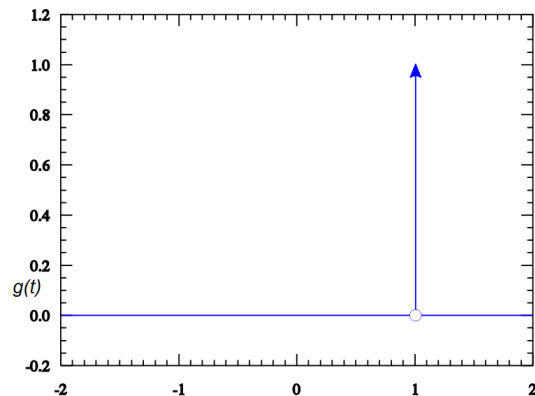
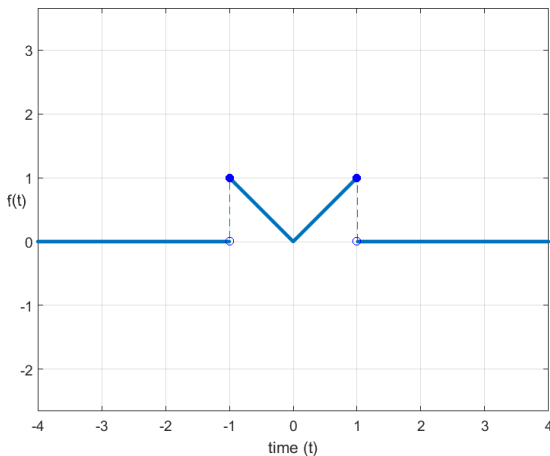
- Do not copy the solutions of your classmates.
- You are encouraged to discuss the problems with your classmates in whatever way you like but make sure to REPRODUCE YOUR OWN SOLUTIONS in what you submit for grading.
- Cite all the online sources that you get help from.
- Keep your work in a secure place.

## Problem 1

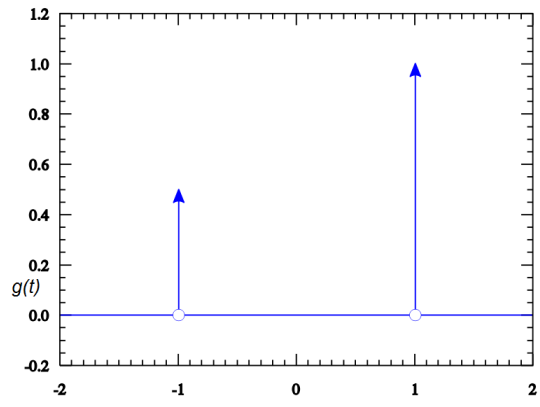
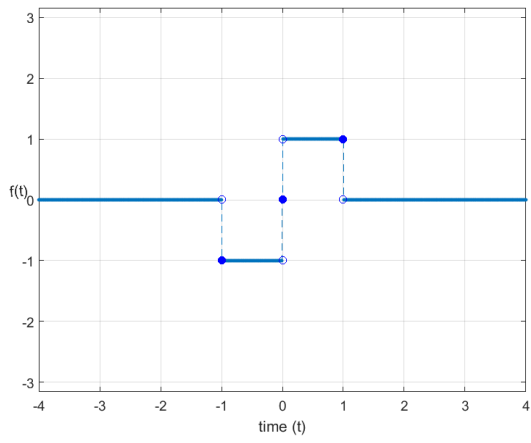
In each case, evaluate the convolution product  $f(t) * g(t)$  and sketch its graph. Write your answer in terms of  $t$  for appropriate range of values of  $t$ .



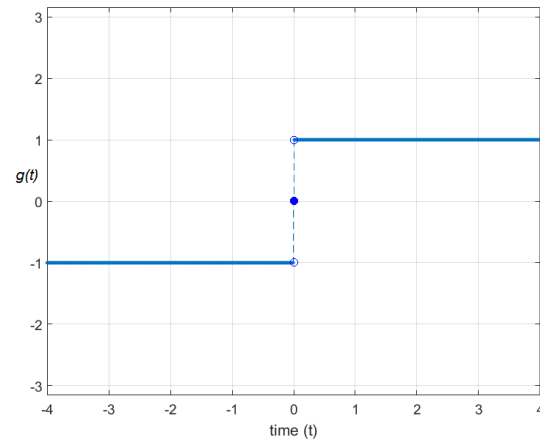
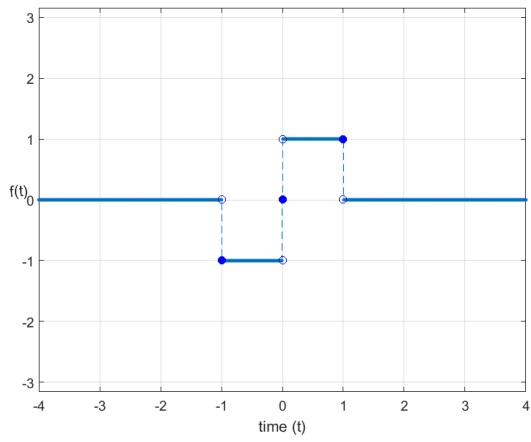
(a)



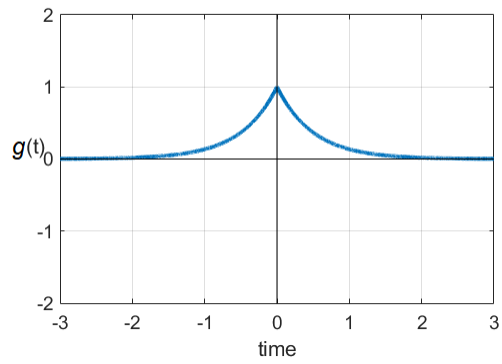
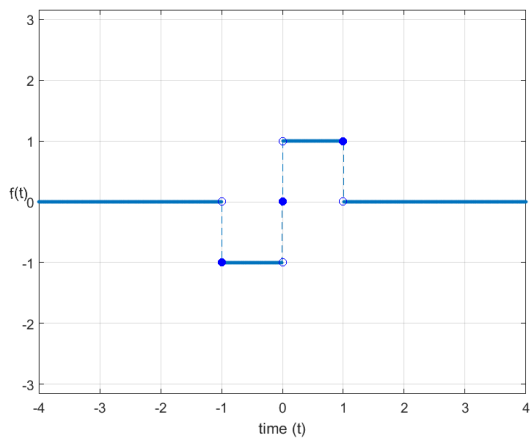
(b)



(c)



(d)



(e)  $g(t) = e^{-|t|}$

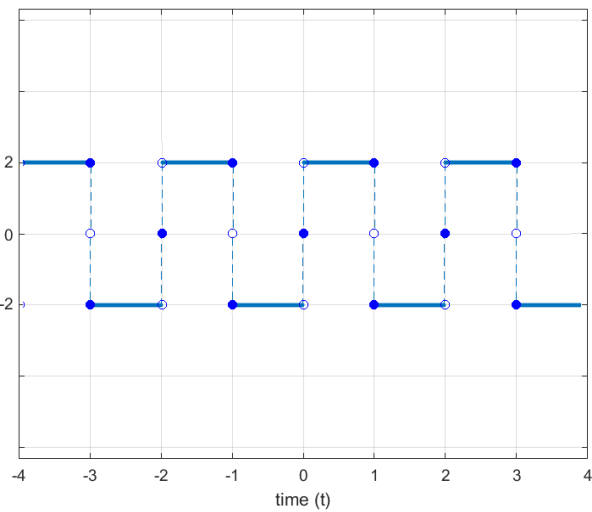
## Problem 2

Use Parseval's theorem (and possibly duality) to evaluate the following integral

$$\int_{-\infty}^{\infty} \frac{\sin^4 2t}{t^4} dt$$

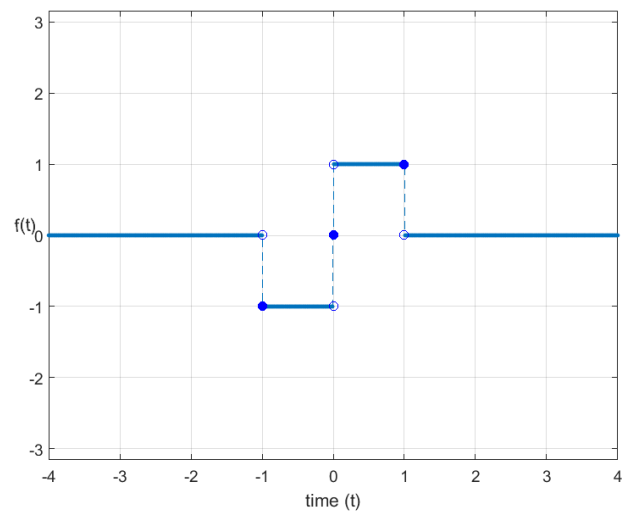
## Problem 3

Find the function  $g(t)$  such that  $f(t) * g(t)$  gives the following square wave,



$$f(t) * g(t)$$

where  $f(t)$  is shown in the following figure. Also sketch the graph of  $g(t)$ .



$$f(t)$$

## Problem 4

Find the Fourier transform of the following functions and sketch their graphs. You may use the transforms table and properties in ‘Notes’ to evaluate the transforms directly.

- (a)  $f(t) = 5e^{-i2t}$
- (b)  $f(t) = \delta(t - 100)$  (Plot the amplitude and phase spectrums)
- (c)  $f(t) = \cos 2t + 3$
- (d)  $f(t) = 1 + 3 \cos 3t + 5 \cos 5t + 7 \cos 7t$
- (e)  $f(t) = \text{sgn}(5t)$  (Plot the amplitude and phase spectrums)
- (f)  $f(t) = \frac{5e^{-i2t}}{t}$  (Plot the imaginary part)

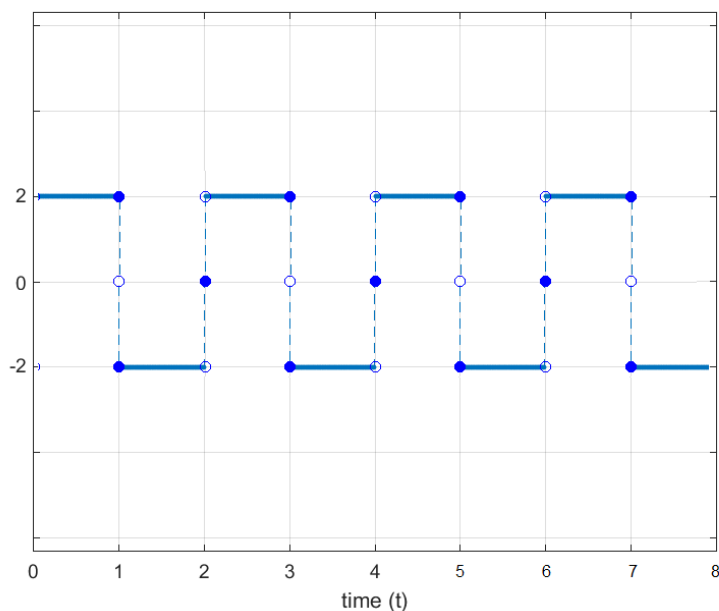
## Problem 5

For each of the following functions, if the function is of ‘exponential order’, find its Laplace transform using transforms table and properties. Also find and sketch the region of absolute convergence in each case. Assume that all the functions are causal i.e.  $f(t) = 0$  for  $t < 0$ .

- (a)  $3e^{5t} \cos 2t$
- (b)  $\int_0^t e^{5\tau} \cos 2\tau d\tau$
- (c)  $(t - 2)^5$
- (d)  $(t - 2)^{-3}$
- (e)  $(t + 2)^{-1}$
- (f)  $\frac{d}{dt}(t + 2)^{-1}$
- (g)  $u(t - 1) - \delta(t - 3)$
- (h)  $u(t - 1) - u(t - 3)$

## Problem 6

Find Laplace transform of the following periodic function.



Periodic function  $f(t)$