Tips to avoid plagiarism

- Do not copy the solutions of your classmates.

- Your are encouraged to discuss the problems with your classmates in whatever way you like but make sure to REPRODUCE YOUR OWN SOLUTIONS in what you submit for grading.

- Cite all the online sources that you get help from.

- Keep your work in a secure place.

Problem 1

For each of the following functions

(a) $f(z) = \dfrac{z}{(z-1)(z-2)(z+2)^2}$

(b) $f(z) = \dfrac{\cos 2z - 1}{2z(z+i)}$

(i) Find all the singular points.

(ii) Classify each singular point as removable or a pole (multiplicity n). Note: A simple pole has multiplicity $n = 1$.

(iii) Find the residue corresponding to each singular point.

(iv) Evaluate the integral $\displaystyle\oint_{|z|=5} f(z)dz$ using residues.
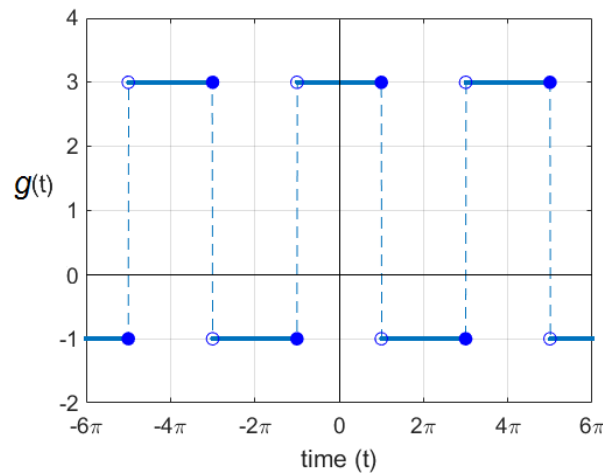
Problem 2

(a) Consider the periodic function shown in the following figure.
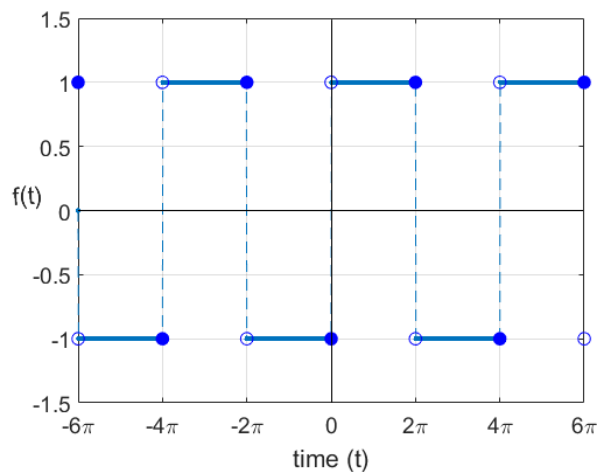
(i) Find its time period T and angular frequency ω_0.

(ii) Is it an even function or an odd function?

(iii) Based on (ii), determine whether its complex Fourier series coefficients c_n be real or pure imaginary? Explain.

(iv) Based on (ii), determine the relationship between c_n and c_{-n}.

(v) Write down the equation of the function in the interval $-\frac{T}{2} < t \le \frac{T}{2}$.

(vi) Now find its complex Fourier series. Clearly write the final expressions for c_0 and c_n.

(vii) Using your answer to (vi), evaluate the Fourier series coefficients a_n and b_n.

(viii) Plot its amplitude spectrum and phase spectrum for $-7 \le n \le 7$.

(ix) Calculate the average power in $f(t)$.

(x) Plot its power spectrum $|c_n|^2$ for $-7 \le n \le 7$.

(b) Now consider the periodic function $g(t)$ shown in the following figure. This function is somehow related to $f(t)$ from part (a).


(i) Find its time period T and angular frequency ω_0.

(ii) Is it an even function or an odd function?

(iii) Based on (ii), determine whether its complex Fourier series coefficients d_n be real or pure imaginary? Explain.

(iv) Based on (ii), determine the relationship between d_n and d_{-n}.

(v) Determine the relationship between $f(t)$ and $g(t)$ and write down an expression for $g(t)$ in terms of $f(t)$. [Hint: $g(t) = af(b(t - t_0)) + c$. Determine a, b, c and t_0.]

(vi) Now using the Fourier series coefficient of $f(t)$, find the complex Fourier series of $g(t)$. Clearly write the final expressions for d_0 and d_n. [Hint: Use properties of Fourier series.]

(vii) Plot its amplitude spectrum and phase spectrum for $-7 \le n \le 7$.

(viii) Calculate the average power in $g(t)$.

(ix) Plot its power spectrum $|d_n|^2$ for $-7 \le n \le 7$.

Problem 3

Repeat Problem 2 for following functions $f(t)$ and $g(t)$.

(a) Function $f(t)$ with complex Fourier series coefficients c_n.
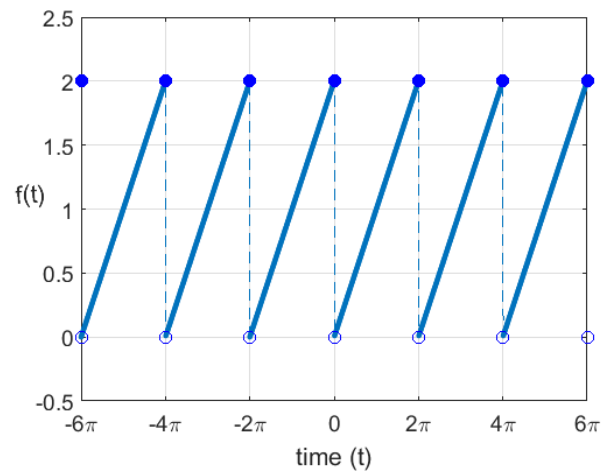
(b) Function $g(t)$ with complex Fourier series coefficients d_n.