MT240: Complex Variables and Transforms

Final Exam (Spring 2019)

Wednesday, May 22

Name: _____ Roll Number: _____

180 Minutes

Instructions

- There are **20** printed pages and **6** blank page in this booklet.
- **All problems** are compulsory.
- **Calculators** are strictly **not allowed**.
- Write **all your work in this booklet**, including any rough work.
- Read the statement **carefully** before you start attempting a problem.
- Properly **label all the axes and relevant points** if you draw any graphs.
- You are allowed to get help from **your own hard copy of lecture notes** uploaded on Google Classroom.
- This exam will assess your following **Course Learning Objectives** (CLOs)
  - CLO 1: Determine whether a complex function is analytic.
  - CLO 2: Calculate the mapping through a complex analytic function.
  - CLO 3: Evaluate the integrals related to Fourier and Laplace transforms for standard functions and interpret their graphs.

Problem	**1**	**2**	**3**	**4**	**5**	**Total**
Marks	15	30	25	20	10	**100**
	CLO 1	CLO 3				

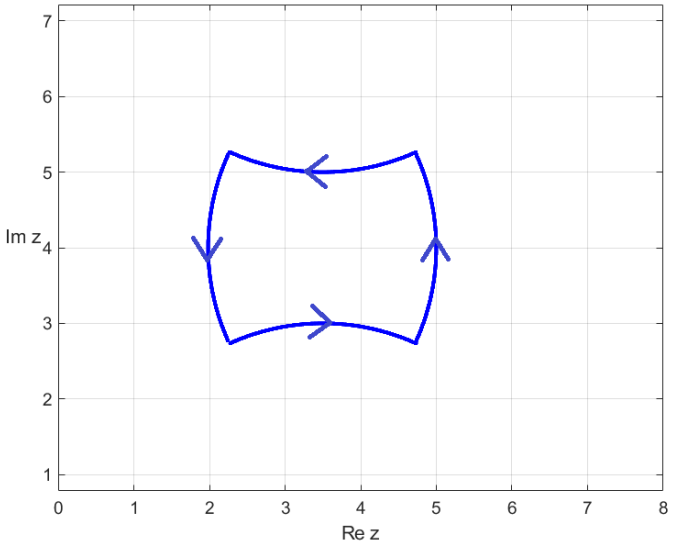
Course Instructor: Usama Bin Sikandar

Page for marks and contestation.
Do NOT write anything on this page.

P1	P2	P3	P4	P5	Total
15	**30**	**25**	**20**	**10**	**100**

Problem 1 [15 marks]

Consider a closed contour γ shown in the figure below.



Evaluate the following integrals.

(a) $\displaystyle\oint_\gamma \frac{z-3-4i}{z}\,dz$

(b) $\displaystyle\oint_\gamma \frac{z}{z-3-4i}\,dz$

(c) $\displaystyle\oint_\gamma \frac{z^2}{(z-3-4i)^2}\,dz$

(d) $\displaystyle\oint_\gamma \frac{z}{(z-4-4i)(z-3-4i)^2}\,dz$

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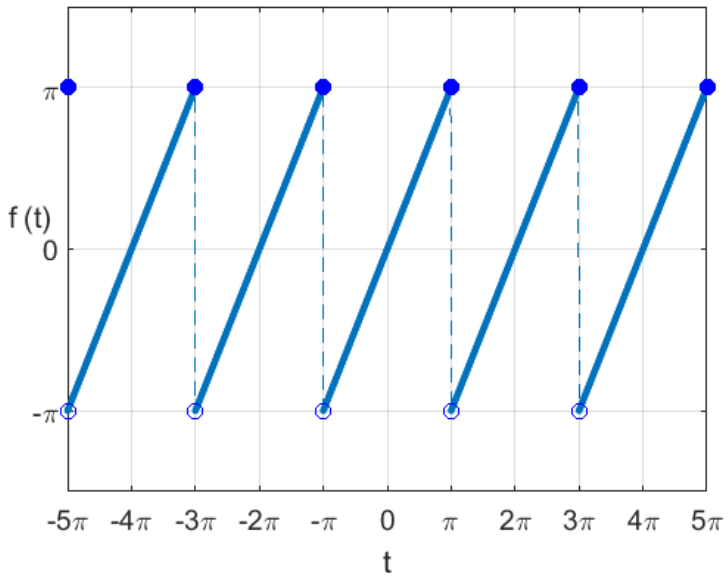
Problem 2 [30 marks]

(a) Consider the periodic function $f(t)$ shown in the figure.



(i) What is the time period of $f(t)$?

(ii) Is the function even or odd? Explain.

(iii) Based on (ii), which of the following is true about the complex Fourier series coefficients of $f(t)$?

☐ $c_n = c_{-n}$

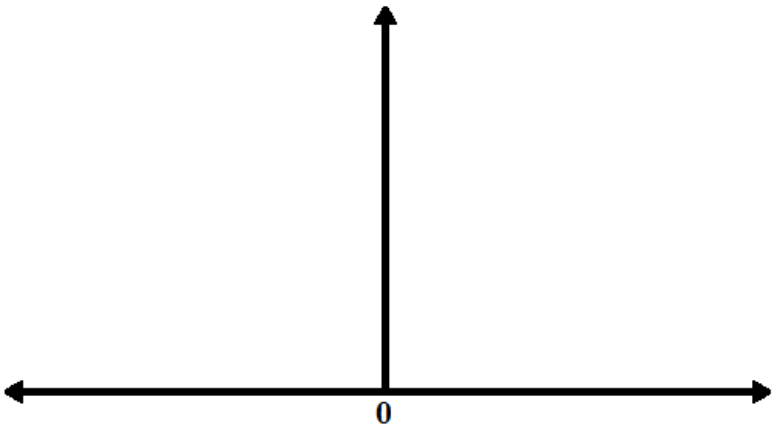
☐ $c_n = -c_{-n}$

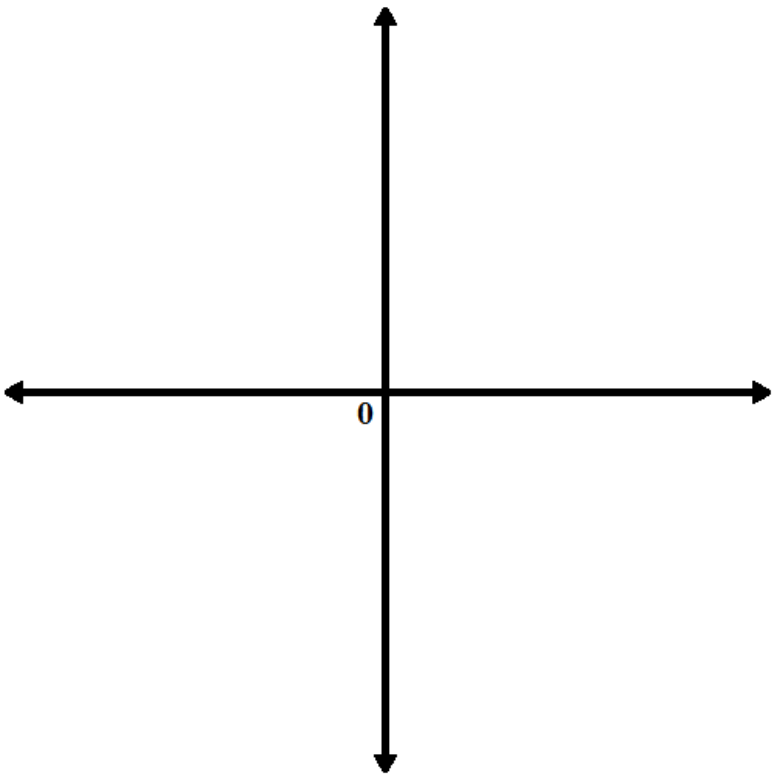
(iv) Write down an expression for $f(t)$ in terms of t for $-\frac{T}{2} < t \leq \frac{T}{2}$.

(v) Evaluate its complex Fourier series coefficients c_n.

(vi) Using your answer to (v), find a_n and b_n, the coefficients of Fourier cosine series and sine series respectively.

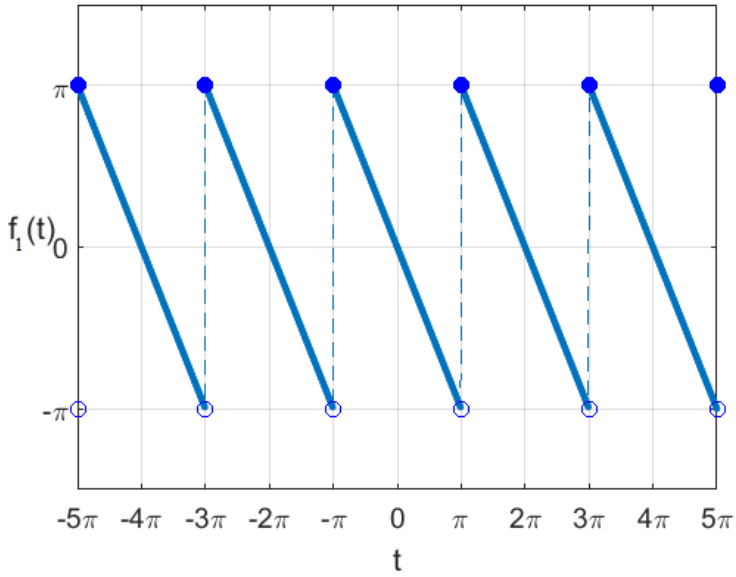
(vii) Plot the amplitude spectrum $|c_n|$ and phase spectrum $\angle c_n$ on the axes given below for $n = [-4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4]$.



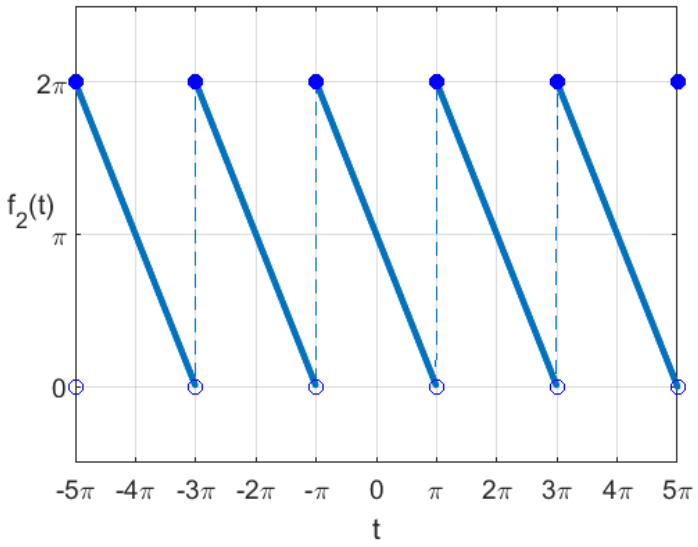


(viii) Use Parseval's identity to evaluate the summation $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

(b) Using your answer to a(v), find the complex Fourier series coefficients d_n of the following function $f_1(t)$.



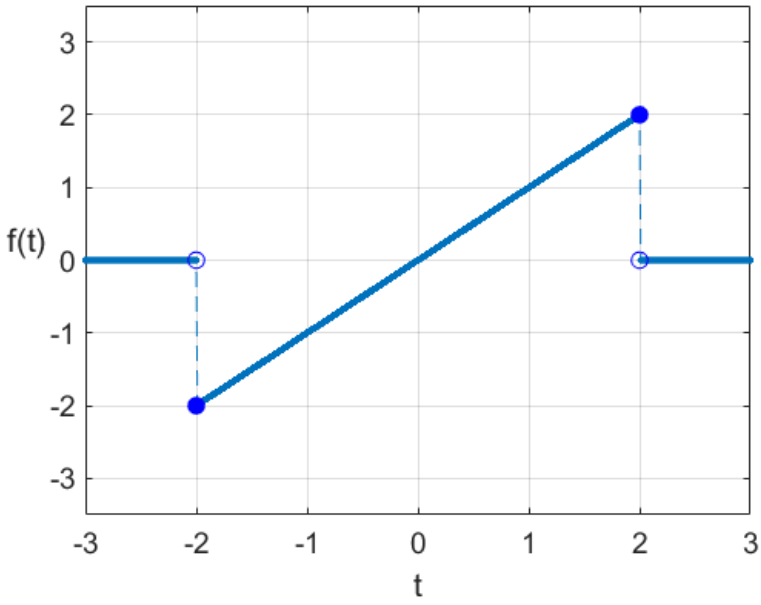
(c) (i) Using your answer to a(v), find the complex Fourier series coefficients e_n of the following function $f_2(t)$.



(ii) Find the value of Fourier series of $f_2(t)$ at $t = -\frac{3}{2}T$.

Problem 3 [25 marks]

Consider the function $f(t)$ shown in the figure below.

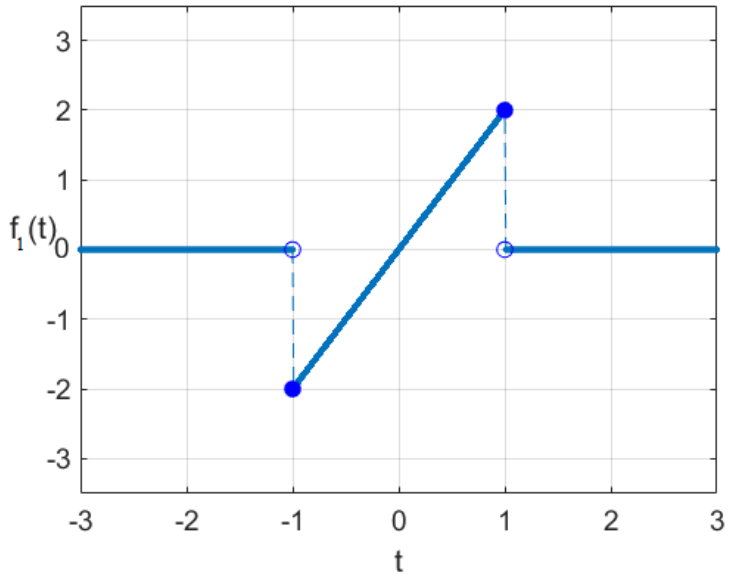


(a) Is this function even or odd? Explain.

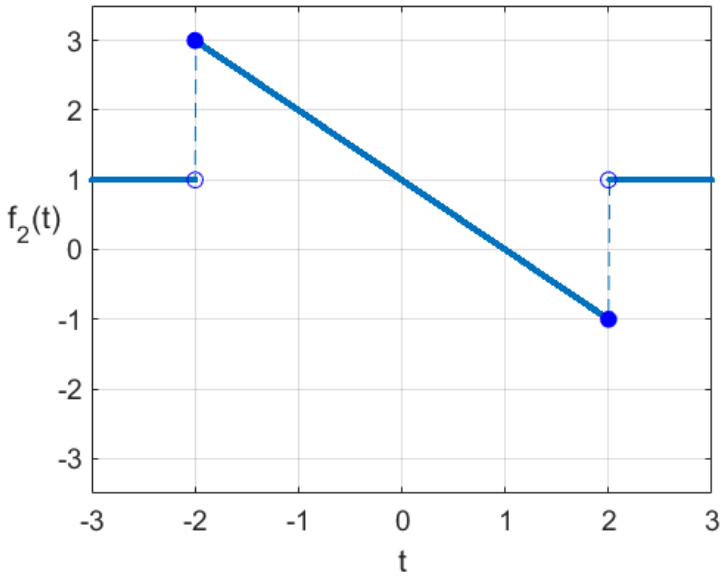
(b) Based on (a), is its Fourier transform $F(\omega)$ real or pure imaginary?

(c) Evaluate $F(\omega)$.

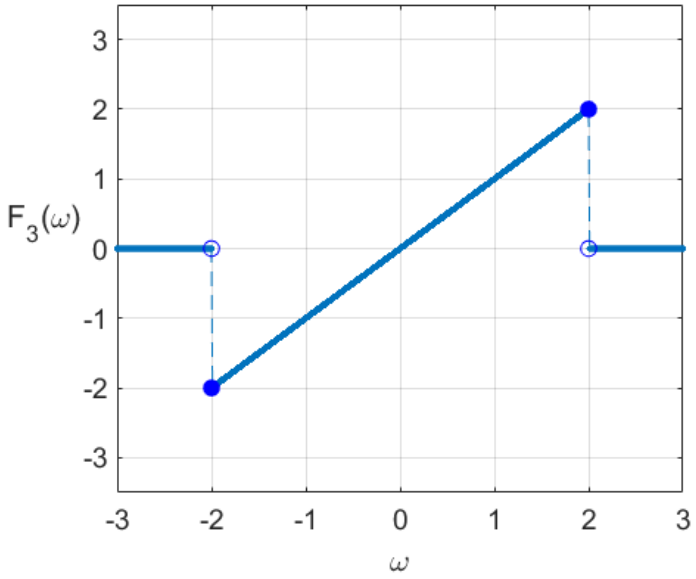
(d) Using your answer to (c), find the Fourier transform of the following function $f_1(t)$.



(e) Using your answer to (c), find the Fourier transform of the following function $f_2(t)$.



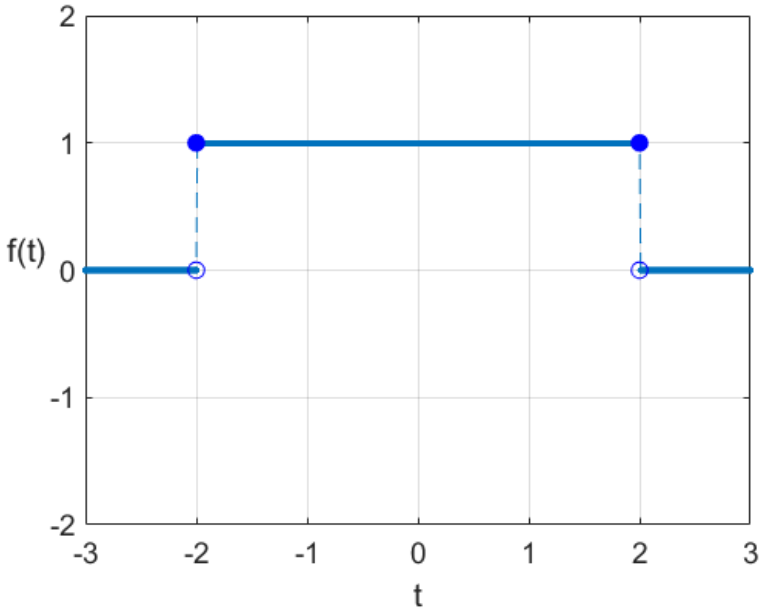
(f) The Fourier transform of a function $f_3(t)$ is shown in the figure below. Using your answer to (c), find the function $f_3(t)$.



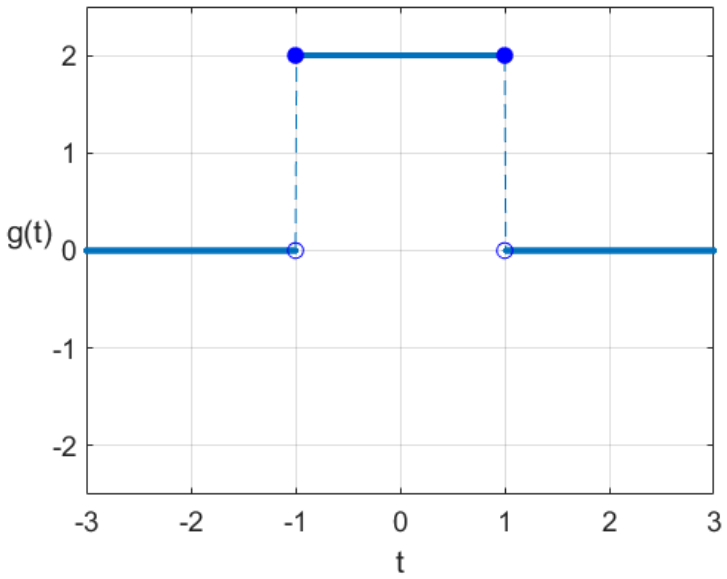
(g) Using the shape of $F_3(\omega)$, determine whether $f_3(t)$ is absoltely integrable.

Problem 4 [20 marks]

Consider the function $f(t)$ shown in the figure.



(a) Evaluate the convolution product $f(t) * g(t)$, where $g(t)$ is shown below.



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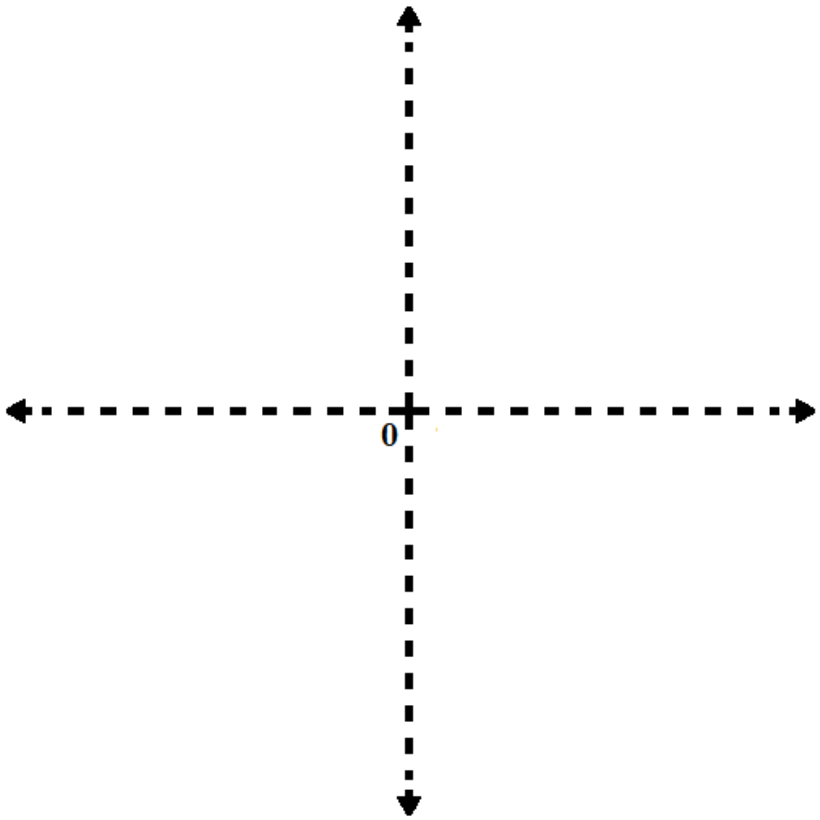
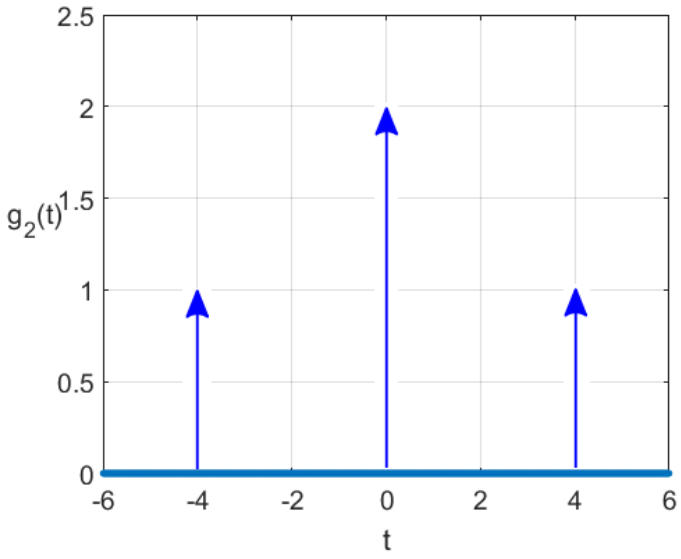
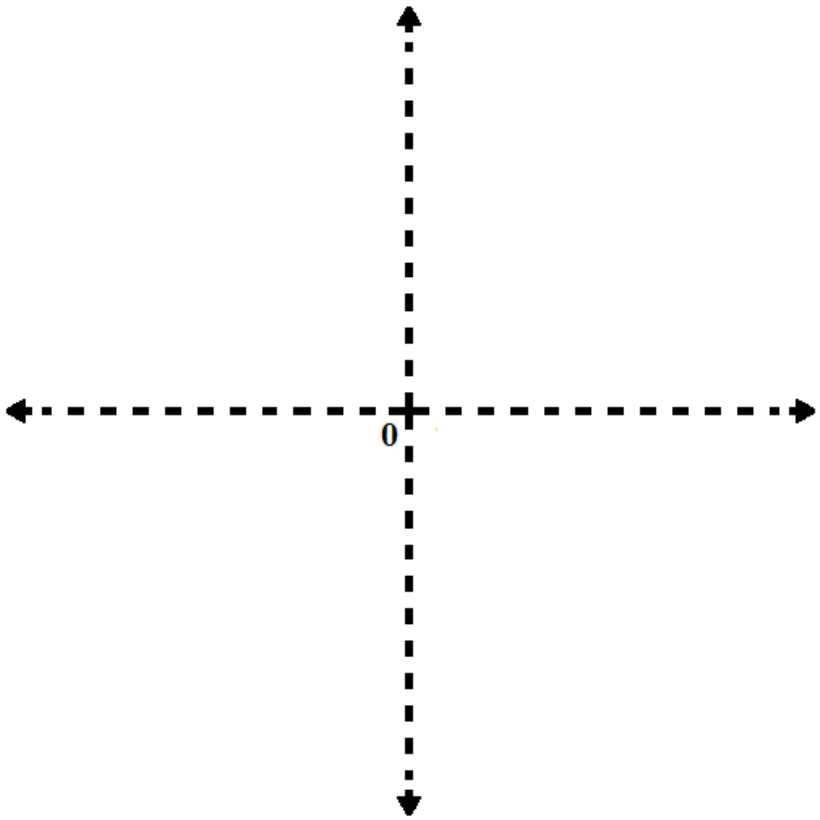
(b) Sketch the graph of $f(t) * g(t)$.



(c) Without evaluating any integral, find the Fourier transform of $f(t) * g(t)$.

(d) Sketch the graph of convolution product $f(t) * g_2(t)$, where $g_2(t)$ is shown below.

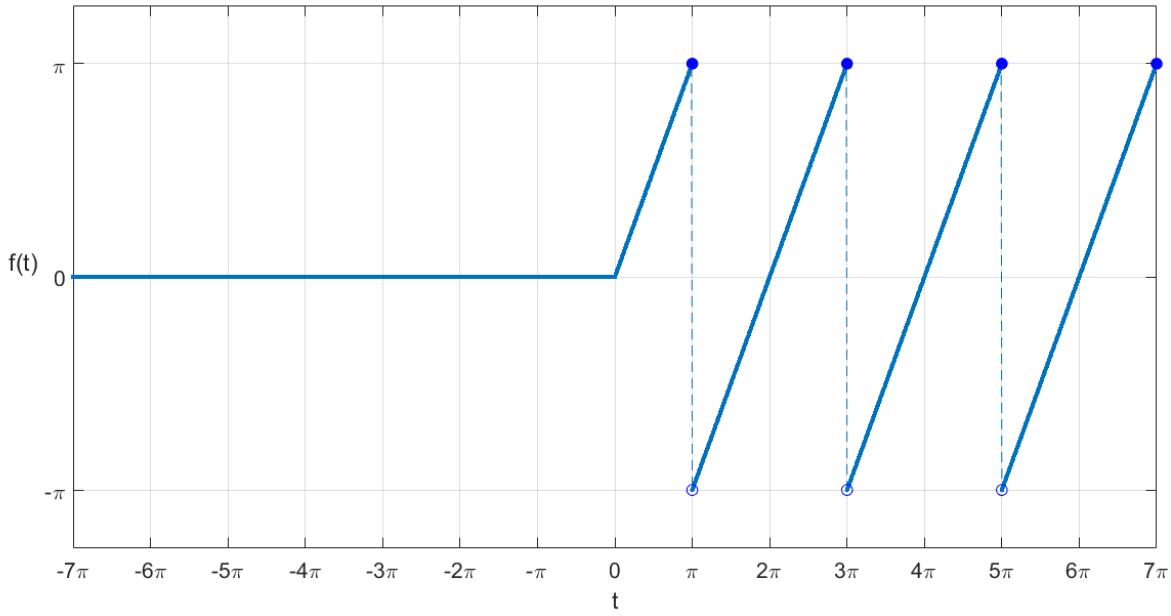




(e) Without evaluating any integral, find the Fourier transform of $f(t) * g_2(t)$.

Problem 5 [10 marks]

Evaluate the Laplace transform of the following function $f(t)$ and find its region of convergence. Notice that the function is periodic for $t \geq 0$ but is equal to 0 for $t < 0$.



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