MT232: Differential Equations

Worksheet 7

Fri Oct 19, 2018

Fall 2018

Student 1 Roll#____________________ **Evaluator 1 Roll#**____________________

Student 2 Roll#____________________ **Evaluator 2 Roll#**____________________

Problem 1 $[10 + 10 + 15 + 10 + 5 = 50]$

Consider the differential equation given below:

$$y' = 2x + y, \; y(0) = 0$$

(a) You need to approximate up to four decimal places the value of the function y with a step size of 0.1. Show your working to approximate the value of $y(0.2)$ using Euler's approximation.

(b) Use the midpoint (RK2) method to approximate up to four decimal places the value of $y(0.2)$ using a step size of 0.1.

(c) Find the analytic solution of differential equation to evaluate the exact values of $y(0.1)$ and $y(0.2)$.

(d) Fill in the following table with the values you found in the previous parts.

Step#	x_i	Analytic $y(x_i)$	Euler's y_i	RK2's y_i	Euler's $\lvert y(x_i) - y_i \rvert$	RK2's $\lvert y(x_i) - y_i \rvert$
0						
1						
2						

(e) The overall absolute error is defined as:

$$\text{Absolute error} = \frac{1}{n+1} \sum_{i=0}^{n} |y(x_i) - y_i|$$

where $y(x_i)$ is an actual value of y at x_i, and y_i is the estimated value at the i^{th} iteration step.

Compute the overall absolute errors for both Euler's and RK2 methods and compare the accuracy of these methods.

Problem 2 $[10 + 10 + 10 = 30]$

Consider the initial value problem given below.

$$\frac{2ty}{t^2+1} - 2t - (2 - \ln(t^2+1))y' = 0, \; y(5) = 0$$

(a) We will use the `ode45()` function in MATLAB to approximate the value of $y(1000)$. It uses a Runge-Kutta 4 (RK4) method to solve differential equations. The function's syntax is given as

```
[t,y] = ode45(odefun, tspan, y0)
```

where `t` and `y` are output arguments, whereas `odefun`, `tspan` and `y0` are input arguments.

The input argument `odefun` specifies the differential equation's function $y'(t) = f(t, y)$ represented in its normal form.

The input argument `tspan` is the solution interval $[t_0 \;\; t_f]$ specifying its starting and ending points, and `y0` is the value of the initial condition $y(t_0)$.

Each value in the solution column vector `y` corresponds to a value returned in the column vector `t` at each iteration step. We will not specify any step size and let the function `ode45()` choose its own optimum step size.

Copy the following code in an .m file to solve the equation using RK4 method. Fill in the two blanks `(...)` with an appropriate code as directed.

```
tspan = [5 1000];
y0 = (...); [insert the value y(t0) in the blank]
[t,y] = ode45(@(t,y) (...)   , tspan, y0); [insert the expression of f(t,y) in the blank]
plot(t,y);
```

Locate the estimate of $y(1000)$ on the graph and find its precise value by clicking at this point after selecting the '+' sign data cursor from the toolbar in the plot window.

Draw a sketch of this graph on your paper, marking all the crucial points.

(b) Find the exact solution of the given differential equation using an analytic method.

(c) Now plot the exact solution in MATLAB and compare with RK4 method. Copy the following code in the same .m file to plot the exact solution in red color. Fill in the blank (...) with an appropriate code as directed.

```
hold on;
t = 5:0.0001:1000;
y = (...);        [insert the expression of the exact solution y(t) in the blank. To apply
                   multiplication, division or exponent operators on arrays, use dot operators
                   '.*', './' and '.^' instead of simple '*', '/' and '^'.]
plot(t,y,'r');
hold off;
```

Locate $y(1000)$ on the graph and find its precise value by clicking at this point after selecting the '+' sign data cursor from the toolbar in the plot window.

Draw a sketch of this graph on your paper, marking all the crucial points.