Student 1 Roll No._____   **Evaluator 1 Roll No._____**

Student 2 Roll No._____   **Evaluator 2 Roll No._____**

Problem 1 (8+8=16 Marks)

Find the general solution of the following Bernoulli Differential Equations.

(a) $xy'(x) + y = \dfrac{1}{y^2}$

(b) $y'(x) - y = e^{2x}y^3$

Problem 2 (1+1+2+10+2=16 Marks)

Let $P(t)$ be the population of a certain animal species. Assume that $P(t)$ satisfies the logistic growth equation

$$\frac{dP}{dt} = 0.2P\left(1 - \frac{P}{200}\right), \qquad P(0) = 150$$

(a) Is the differential equation autonomous?

(b) Is the differential equation linear?

(c) Without solving the differential equation, give a sketch of the graph of $P(t)$.

(d) This differential equation can be transformed into a linear differential equation through a suitable substitution, and then it can be solved using an integrating factor. Solve the differential equation again, using this method.

(e) Sketch $P(t)$ found in (d).

Problem 3 (8+8=16 Marks)

For the following Differential Equations; a) Determine whether the given DE is exact or not. b) If the DE is exact, solve it.

(a) $4x\sin(y)dx + 2x^2\cos(y)dy = 0$

(b) $3x^2ydx + x^3ydy = 0$

Problem 4 (16 Marks)

In the following problems find the value of k so that the given DE is exact and solve the respective equation.

(a) $(y^3 + kxy^4 - 2x)dx + (3xy^2 + 20x^2y^3)dy = 0$

Problem 5 (16 Marks)

Show that a one parameter family of solutions of the equation $(4xy + 3x^2)dx + (2y + 2x^2)dy = 0$ is $x^3 + 2x^2y + y^2 = c$