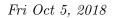
Worksheet 5



## Problem 1

For each of the following differential equation:

- 1. Convert the differential equation into separable form:  $\frac{dy}{dx} = g(x)h(y)$
- 2. Solve the differential equation
- 3. Draw the solution curve for parts (b) and (c)
- (a)  $\frac{dy}{dx} = \frac{x(e^{x^2} + 2)}{6y^2}, \ y(0) = 1$
- (b)  $y' = e^{-y}(2x 4), y(5) = 0$
- (c)  $\frac{dx}{dt} = 4(x^2 + 1), \ x(\pi/4) = 1$
- (d)  $x^2 \frac{dy}{dx} = y xy, \ y(-1) = -1$

## Problem 2

The Gompertz model is  $y' = -Ay \ln y$  (A > 0), where y(t) is the mass of tumor cells at time t. The model agrees well with clinical observations. The declining growth rate with increasing y > 1 corresponds to the fact that cells in the interior of a tumor may die because of insufficient oxygen and nutrients. Use the differential equation to discuss the growth and decline of solutions (tumors) and to find constant solutions. Then solve the differential equation.

## Problem 3

For each of the following differential equation:

- (i) Identify if the differential equation is linear or non-linear
- (ii) Identify if the differential equation is homogeneous or non-homogeneous
- (iii) Solve the linear differential equation clearly mentioning each step

(a) 
$$xy' + y = e^x$$
,  $y(1) = 2$ 

(b)  $L\frac{di}{dt} + Ri = E$ ,  $i(0) = i_0$ . L, R, E,  $i_0$  are constants.

(c) 
$$(x+1)\frac{dy}{dx} + y = \ln x, \ y(1) = 10$$

(d)  $ty' + 2y = t^2 - t + 1$ ,  $y(1) = \frac{1}{2}$ 

## Problem 4

A rock contains two radioactive isotopes,  $RA_1$  and  $RA_2$ , that belong to the same radioactive series, that is,  $RA_1$  decays into  $RA_2$ , which then decays into stable atoms. Assume that the rate at which  $RA_1$  decays into  $RA_2$  is  $50e^{-10t}$ kg/sec. Because the rate of decay of  $RA_2$  is proportional to the mass of  $RA_2$  present, the rate of change in  $RA_2$  is

$$\frac{dy}{dt} = 50e^{-10t} - ky$$

where k > 0 is decay constant. If k = 2/sec and initially y(0) = 40kg.

- 1. Find the mass y(t) of  $RA_2$  for  $t \ge 0$ .
- 2. Plot the mass calculated in (a).