MT232: Differential Equations

Worksheet 5

Fri Oct 5, 2018

Fall 2018

Problem 1

For each of the following differential equation:

1. Convert the differential equation into separable form: $\dfrac{dy}{dx} = g(x)h(y)$
2. Solve the differential equation
3. Draw the solution curve for parts (b) and (c)

(a) $\dfrac{dy}{dx} = \dfrac{x(e^{x^2}+2)}{6y^2},\ y(0) = 1$

(b) $y' = e^{-y}(2x-4),\ y(5) = 0$

(c) $\dfrac{dx}{dt} = 4(x^2+1),\ x(\pi/4) = 1$

(d) $x^2\dfrac{dy}{dx} = y - xy,\ y(-1) = -1$

Problem 2

The Gompertz model is $y' = -Ay\ln y$ $(A > 0)$, where y(t) is the mass of tumor cells at time t. The model agrees well with clinical observations. The declining growth rate with increasing $y > 1$ corresponds to the fact that cells in the interior of a tumor may die because of insufficient oxygen and nutrients. Use the differential equation to discuss the growth and decline of solutions (tumors) and to find constant solutions. Then solve the differential equation.

Problem 3

For each of the following differential equation:

(i) Identify if the differential equation is linear or non-linear

(ii) Identify if the differential equation is homogeneous or non-homogeneous

(iii) Solve the linear differential equation clearly mentioning each step

(a) $xy' + y = e^x,\ y(1) = 2$

(b) $L\dfrac{di}{dt} + Ri = E,\ i(0) = i_0$. L, R, E, i_0 are constants.

(c) $(x+1)\dfrac{dy}{dx} + y = \ln x,\ y(1) = 10$

(d) $ty' + 2y = t^2 - t + 1,\ y(1) = \dfrac{1}{2}$

Problem 4

A rock contains two radioactive isotopes, RA_1 and RA_2, that belong to the same radioactive series, that is, RA_1 decays into RA_2, which then decays into stable atoms. Assume that the rate at which RA_1 decays into RA_2 is $50e^{-10t}$kg/sec. Because the rate of decay of RA_2 is proportional to the mass of RA_2 present, the rate of change in RA_2 is

$$\frac{dy}{dt} = 50e^{-10t} - ky$$

where $k > 0$ is decay constant.If $k = 2/sec$ and initially $y(0) = 40$kg.

1. Find the mass $y(t)$ of RA_2 for $t \geq 0$.
2. Plot the mass calculated in (a).
