Worksheet 4

Fri Sep 28, 2018

Fall 2018

Problem 1

Find all the equilibrium points of the given autonomous first-order differential equations and classify each equilibrium point as either stable, unstable or semi-stable. Also sketch phase portrait for each differential equation.

(a) $y'(x) = 3y$

(b) $x'(t) = 5x + 9$

(c) $y'(t) = y^2 - 3y$

(d) $x'(t) = x(2 - x)(4 - x)$

(e) $y' = y^2 - y^3$

Problem 2

The mathematical formulation of Newton's empirical law of cooling of an object described by

$$\frac{dT}{dt} = k(T - T_m)$$

Suppose that a cup of tea starts out at time $t = 0$ at a temperature of 120 degrees Celsius in a room whose temperature is 60 degrees Celsius and k = - 0.05. Find the equilibrium point and classify it. Also draw a phase portrait.

Problem 3

Let $P(t)$ be a population of a certain animal species. Assume that follows the logistic growth differential equations

$$\frac{dP}{dt} = 0.2P\left(1 - \frac{P}{100}\right)$$

Find the equilibrium points and classify them. Also draw the phase portrait diagram.

Problem 4

When certain kinds of chemicals are combined, the rate at which the new compound is formed is modeled by the autonomous differential equation

$$\frac{dX}{dt} = k(\alpha - X)(\beta - X)$$

where $k > 0$ is a constant of proportionality and $\beta > \alpha > 0$. Here $X(t)$ denotes the number of grams of the new compound formed in time t

(a) Use a phase portrait of the differential equation to predict the behavior of $X(t)$ as $t \to \infty$

(b) Consider the case when $\alpha = \beta$.Use a phase portrait of the differential equation to predict the behavior of X(t) as $t \to \infty$ when $X(0) = \alpha > 0$.