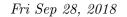
Worksheet 4



Problem 1

Find all the equilibrium points of the given autonomous first-order differential equations and classify each equilibrium point as either stable, unstable or semi-stable. Also sketch phase portrait for each differential equation.

- (a) y'(x) = 3y
- (b) x'(t) = 5x + 9
- (c) $y'(t) = y^2 3y$
- (d) x'(t) = x(2-x)(4-x)
- (e) $y' = y^2 y^3$

Problem 2

The mathematical formulation of Newton's empirical law of cooling of an object described by

$$\frac{dT}{dt} = k(T - T_m)$$

Suppose that a cup of tea starts out at time t = 0 at a temperature of 120 degrees Celsius in a room whose temperature is 60 degrees Celsius and k = -0.05. Find the equilibrium point and classify it. Also draw a phase portrait.

Problem 3

Let P(t) be a population of a certain animal species. Assume that follows the logistic growth differential equations

$$\frac{dP}{dt} = 0.2P\left(1 - \frac{P}{100}\right)$$

Find the equilibrium points and classify them. Also draw the phase portrait diagram.

Problem 4

When certain kinds of chemicals are combined, the rate at which the new compound is formed is modeled by the autonomous differential equation

$$\frac{dX}{dt} = k(\alpha - X)(\beta - X)$$

where k > 0 is a constant of proportionality and $\beta > \alpha > 0$. Here X(t) denotes the number of grams of the new compound formed in time t

- (a) Use a phase portrait of the differential equation to predict the behavior of X(t) as $t \to \infty$
- (b) Consider the case when $\alpha = \beta$. Use a phase portrait of the differential equation to predict the behavior of X(t) as $t \to \infty$ when $X(0) = \alpha > 0$.

