MT232: Differential Equations

Worksheet 2

Fri Sep 14, 2018 *Fall 2018*

Answer all these steps for each problem.

(1) Write down the purpose of the model.

(2) Identify the variables of interest.

(3) Make assumptions about the relationship of variables based on prior knowledge, observations and empirical laws. In mechanical or circuit models, choose a suitable positive direction of motion or current flow.

(4) Convert your assumptions to mathematical equations.

Problem 1

Consider the community of rats, the population growth of rats at a certain time in the community is proportional to the total population of rats present at that time. Let $P(t)$ be the total population of rats at the time t. Derive a simple model which determines the population of the rats in the community.

Problem 2

Let's assume that you are drinking a cup of coffee in cold winter. Suddenly you start thinking that how much time will this cup take to cool down. You know that rate at which the temperature of the cup changes depends on the temperature of the cup and the temperature of surrounding medium. Let $T(t)$ be the temperature of the cup and 15° C is the temperature of the surrounding medium. Find the model equations.

Problem 3

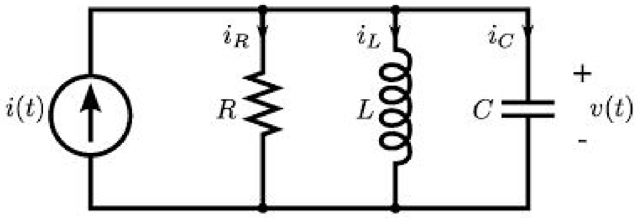
An Ebola virus is spread throughout a community of size n by people coming into contact with other people. The spread of Ebola is proportional to the number of encounters or interactions, between these two groups of people. Let $S(t)$ denote the number of the susceptible peoples and $I(t)$ denote the number of the infected people at the certain time in the community. Write the model equation for number of infected people.

Problem 4

The radioactive isotope Indium-111 is often used for diagnosis and imaging in nuclear medicine. Suppose the mass m of this radioactive isotope indium-111 decays with a decay constant of 3.47. Write the differential equation which describes the rate at which m decays with respect to time.

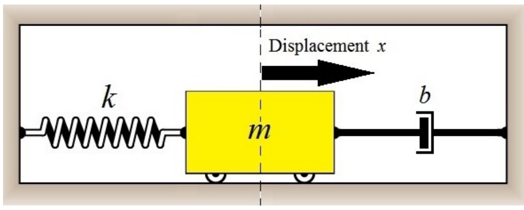
Problem 5

Suppose we have a parallel RLC circuit as shown in figure below. Write the mathematical model of differential equation for the voltage $v(t)$. (Hint: K.C.L.)



Problem 6

Consider a cart of mass m is attached to a spring of spring constant k from one end, and from other end it is connected to a damper with damping constant b, forming a spring-mass-damper system as shown in the figure below. The damper opposes the velocity with which mass m moves in the horizontal direction. Write the mathematical model differential equation for the displacement $x(t)$ when cart moves in horizontal direction as indicated in the figure.



Problem 7

(a) Romeo is in love with Juliet, but in our version of this story, Romeo is a fickle lover. The more Juliet loves him, the more he begins to dislike her. But when she loses interest, his feelings for her warm up. She, on the other hand, tends to echo him: her love grows when he loves her but reduces when he hates her. Write the simple mathematical model equation of their love affair.

(b) Now let's discuss the personalities of Laila and Majnoo. Both of them have a tendency to get over each other as the time passes. Moreover, they are also identically cautious in their relationships: they both get excited by the other person's advances and they both try to avoid throwing themselves at the other. However, their tendency to get over is stronger than their desire for each other. Describe their such behavior in mathematical model.

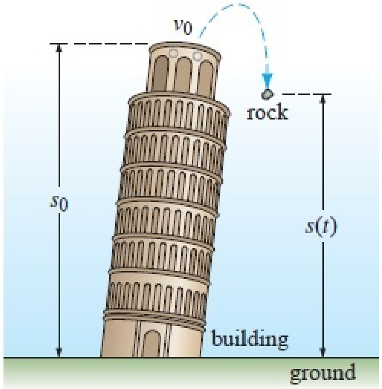
Problem 8

Suppose we have a Chemical Reaction between CH_3Cl and NaOH, which combines in such a way that initial amount of CH_3Cl is α and we have a very large amount of NaOH for chemical reaction that it can never deplete. These two compounds combine to form an X amount of CH_3OH with some amount of NaCl. Write the mathematical model of differential equation for $X(t)$.

$$\mathrm{CH_3Cl + NaOH \longrightarrow CH_3OH + NaCl}$$

Problem 9

A rock is thrown from the top of Pisa tower at the initial velocity v_0. Let $s(t)$ be the relative height of the rock at time t. While falling from building the rock also faces air resistance with c as the drag constant. Derive a differential equation for the height of the rock.



Problem 10

Suppose that a water tank of cross-sectional area $A_w = 5\ \text{m}^2$ has two holes with areas $A_{h1} = 5\ \text{cm}^2$ and $A_{h2} = 3\ \text{cm}^2$ at its bottom and that water is draining from the holes. Let $h(t)$ and $V(t)$ denote the depth and the volume of water in the tank at time t (in seconds).

(a) Find the velocity u of the stream of water exiting the tank from each hole.

(b) At what rate the depth of water changes with time, write this scenario in a mathematical model.