MT232: Differential Equations

Worksheet 1

Fri Sep 7, 2018

Fall 2018

Problem 1

Determine domain and range for each of the following functions. Also sketch the graphs of these functions by hand.

(a) $3x - 2y = -4$

(b) $2y + 3x = 8$

(c) $y = x^2 - 8x + 16$

(d) $y = -x^2 + 10x - 22$

(e) $y = 31e^{-2x}$

(f) $y = 17e^{\frac{1}{2}x}$

(g) $y = \cos 2x$

(h) $y = 2\sin x$

Problem 2

Find the equation of a straight line that

(a) passes through the point $(-1, -2)$ and has a gradient of $\frac{4}{7}$.

(b) joins the points $(3, 6)$ and $(2, -5)$.

Also sketch these lines.

Problem 3

You opened your differential equations book, and noticed that the product of the two pages in front of you was equal to 1122. What were the numbers of those pages?

Problem 4

We discussed in the lecture that any quadratic equation with constant real coefficients

$$f(x) = ax^2 + bx + c,$$

can be converted into the form

$$f(x) = a(x - k)^2 + m,$$

by completing the square. Consider the following functions,

$f(x) = x^2 - 10x - 2$

Sketch the curve, clearly labeling the coordinates of y-intercept.

Problem 5

(a) Plot the following intervals on the real number line. Also specify whether the interval is open or closed.

(i) $\{x : 2 < x \leq 5\}$

(ii) $\{x : -3 < x < -1\}$

(b) Plot the following regions on Cartesian plane. Also specify whether the region is open or closed.

(i) $\{(x, y) : -2 < x \leq 5 \ \wedge \ -5 < y < -1\}$

(ii) $\{(a, b) : a^2 + b^2 \leq 16\}$

Problem 6

Consider the function $f(x) = x^2$.

(a) Calculate the derivative of this function $\dfrac{df}{dx}$.

(b) Compute $\left.\dfrac{df}{dx}\right|_{x=2}$

(c) Now we want to estimate $\left.\dfrac{df}{dx}\right|_{x=2}$ using the formal definition of derivative. The formal definition of derivative is given by

$$\frac{df}{dx}(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x}$$

Fill in the table below:

x	x_o	$f(x) = x^2$	$\Delta f = x^2 - x_o^2$	$\Delta x = x - x_o$	$\dfrac{\Delta f}{\Delta x}$	$\dfrac{df}{dx}(x_o)$
2.5	2					
2.4	2					
2.3	2					
2.2	2					
2.1	2					
2.05	2					
2.01	2					
2.001	2					

What do the values in the table tell you about the estimate of the derivative from its formal definition?