Student 1 Roll No._____   **Evaluator 1 Roll No.**_____

Student 2 Roll No._____   **Evaluator 2 Roll No.**_____

Problem 1 (20+20+20+20 Marks)

For each of the following autonomous systems of linear constant-coefficient homogeneous differential equations,

(a) $x' = -x$
$y' = x - 2y$
$x(0) = 1, \; y(0) = -1$

(b) $x' = x$
$y' = x + 2y$
$x(0) = 1, \; y(0) = -1$

(c) $x' = x + 2y$
$y' = 3x + 2y$
$x(0) = 0, \; y(0) = -4$

(d) $x' = x - 2y$
$y' = 5x - y$
$x(0) = 2, \; y(0) = -5$

(i) Find its general solution.

(ii) Sketch its general phase-portrait of the system, and show at least 8 trajectories in the phase-plane for $t \in (-\infty, \infty)$ starting from different initial conditions of your choice all over the plane (this is how we made those exotic sketches in class).

(iii) Classify its equilibrium point as node, saddle, ellipse or spiral, and also comment on its stability.

(iv) Now for $t \in (0, \infty)$, sketch the trajectory of its solution in the phase-plane that starts from the given initial condition at $t = 0$.

(v) Now use the given initial conditions to find its solution (i.e. evaluate c_1 and c_2).