MT232: Differential Equations

Homework 5

Due Fri Dec 7, 9:00 AM

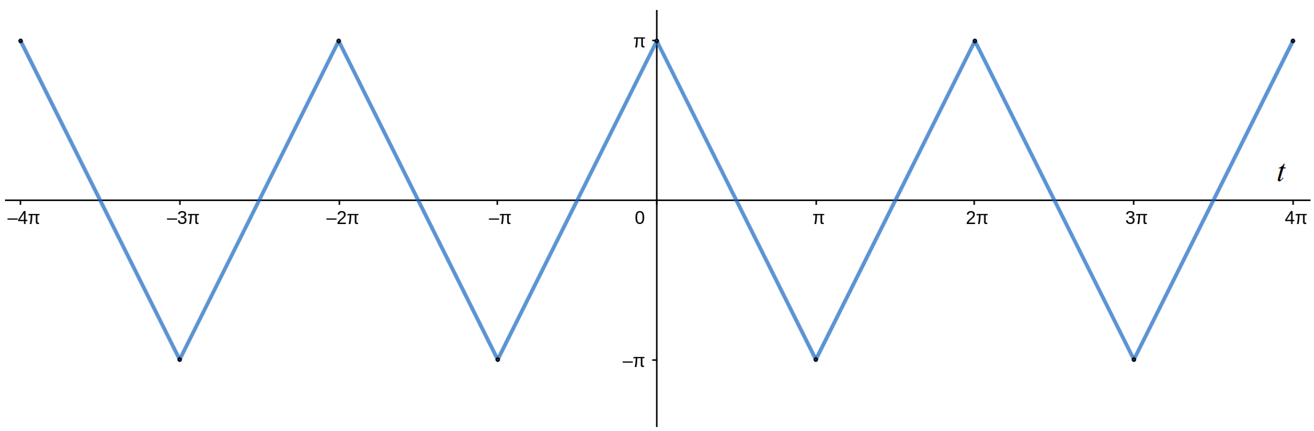
Fall 2018

Tips to avoid plagiarism

- Do not copy the solutions of your classmates.
- Your are encouraged to discuss the problems with your classmates in whatever way you like but make sure to REPRODUCE YOUR OWN SOLUTIONS in what you submit for grading.
- Cite all the online sources that you get help from.
- Keep your work in a secure place.

Problem 1

(a) Consider the triangle wave given below.



Periodic Function $f_1(t)$

(i) Determine its time period T, frequency f and angular frequency ω.

(ii) Write down its expression in the intervals $-\frac{T}{2} < t \leq 0$ and $0 < t \leq \frac{T}{2}$.

(iii) Is this an even function or an odd function? Explain.

(iv) Find its Fourier series.

(v) Evaluate the Fourier series coefficients from $n = 0$ to $n = 9$.

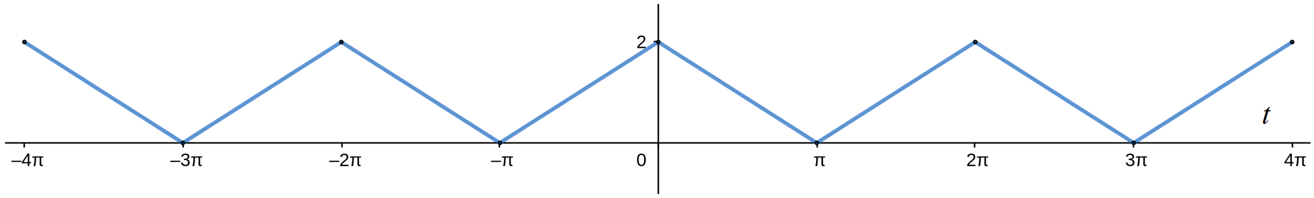
(vi) Find the particular solution of the following differential equation, if $f(t) = f_1(t)$ is the input function of the following differential equation.

$$y'' + 25y = f(t)$$

(vii) What is the resonant frequency of the system in (vi)?

(viii) Would any frequency component of $f_1(t)$ cause resonance in the system? If yes, express that resonant component of the output in terms of t, demonstrating that its amplitude is increasing indefinitely.

(b) Now consider the triangle wave given below.



Periodic Function $f_2(t)$

Repeat all the steps (i)-(viii) in (a) for the triangle wave given in this figure. Use $f(t) = f_2(t)$ for this part. In part (iv), use the Fourier series of $f_1(t)$ to compute the Fourier series of $f_2(t)$, i.e. do not use the Fourier coefficient formulas to evaluate the Fourier series.

Problems 2

Solve the following differential equations by any method of your choice (complexification, undetermined coefficients, variation of parameters or any combination thereof).
Hint: Concepts of linearity and superposition will be helpful.

(a) $\frac{1}{4}y'' + y' + y = x^2 - 2x$

(b) $y'' - y' - 6y = e^{-2x}\sin 2x + 12x$

(c) $y'' + 3y' + 2y = \dfrac{1}{1+e^x}$

(d) $2y'' + 18y = 6\tan 3t$

Problems 3

(a) Compute the Fourier series of the periodic function with a period of 2π and is given by $f(t) = t/2$ for $-\pi < t < \pi$, by evaluating the integral expressions for a_n and b_n (or using even/odd to decide that one or another of these sets of numbers are all zero).

(b) Go to the following mathlet link:
http://mathlets.org/mathlets/fourier-coefficients/
Press “Formula” to see the significance of the sliders. Move them around a bit and watch what happens. The yellow curve gives the sum, the white curve gives the sinusoidal function you are adding to the sum at the moment.

(i) With the settings on “Sine Series” and “All terms,” select target A. Move the sliders around till you get the best fit you can eyeball. Record your results: $b_1 = \ldots$, $b_2 = \ldots$, ...

(ii) Now select target D and do the same. But then, before you record your results, select “Distance.” This makes a number appear above the graph, which gives a measure of the goodness of fit of the partial Fourier series you have built. Move the sliders from the top one to the bottom one to get the best fit you can. Record the results. Notice that you began with large period and then worked your way down to small period.
Now press “Reset,” and do the same thing from the bottom up: you are putting in the best possible multiples of $\sin(6t)$, then $\sin(5t)$, and so on, in that order. Are the numbers you obtain the same as the ones you got going in the other direction? How do these values match up with what you computed in Part (a)? Do you suppose you would get different answers if you put in terms in some other more random order?

Problem 4

Consider the differential equation

$$4x^2y''(x) + 17y(x) = \frac{1}{\sqrt{x}}$$

defined for $x \geq 1$ with initial conditions $y(1) = -1$ and $y'(1) = -\frac{1}{2}$. The homogeneous solutions are given as

$$y_1(x) = \sqrt{x}\cos(2\ln x),$$
$$y_2(x) = \sqrt{x}\sin(2\ln x).$$

(a) Show that y_1 and y_2 are linearly independent.

(b) Find the values of x for which Wronskian $W = 0$.

(c) Using variation of parameters, find the particular solution y_p for Wronskian $W \neq 0$.

(d) Find the overall solution $y(x)$.

Problems 5

Find a solution to the following differential equations:

(a) $x^2y'' + 10xy' + 6y = \ln x^2$

(b) $x^2y'' - 3xy' + 13y = 4 + 3x$
