MT232: Differential Equations

Homework 3

Due: Fri, Oct 26, 9:00 AM

Fall 2018

Tips to avoid any cases of plagiarism

- You must NOT look at the solutions of your classmates.
- Your are encouraged to discuss the homework with your classmates, but restrict yourself to oral discussions only.
- Cite all the online sources that you get help from.
- Keep your work in a secure place.

Problem 1

Find the derivative df of each of the following functions.

(a) $f(x) = x^2 + \cos\sqrt{x}$

(b) $f(x, y) = x^3 + 3x^2y + 3xy^2 + y^3$

(c) $f(x, y, z) = x\cos(yz) - \sin(xz) - z\ln(xy)$

Problem 2

Solve the following initial value problems.

(a) $x\dfrac{dy}{dx} + y = \dfrac{1}{y^2}$, $y(1) = 2$

(b) $(y^2\cos x - 3x^2y - 2x)dx + (2y\sin x - x^3 + \ln y)dy = 0,\ y(0) = e$

(c) $\dfrac{dy}{dx} = \dfrac{(x+y)^2}{1 - 2xy - x^2}$, $y(1) = 1$

(d) $x\dfrac{dy}{dx} + y = y^2x^2\ln x,\ y(1) = -1$

Problem 3

The differential equation $\dfrac{dy}{dx} = P(x) + Q(x)y + R(x)y^2$ is known as Riccati equation has a variety of applications in physics and engineering such as supersymmetric quantum mechanics, variational calculus, nonlinear physics, optimal control systems and thermodynamics. The Riccati equation can be solved by two successive substitution given we know a solution. If y_1 is a solution, the first substitution $y = y_1 + u$ reduces Riccati equation to a Bernoulli equation. Find one parameter family of solutions for the following differential equation:

$$\frac{dy}{dx} = -\frac{4}{x^2} - \frac{1}{x}y + y^2$$

where $y_1 = \dfrac{2}{x}$ is a known solution of the equation.

Problem 4

Consider the following initial value problem.

$y' = ty^2 - \frac{y}{t},\ y(1) = 1$

(a) Solve the differential equation to find its closed-form solution $y(t)$.

(b) Use Euler's method to obtain a four decimal-place approximation of $y(1.2)$ using

  (i) a step size of 0.1

  (ii) a step size of 0.05

(c) Now use the midpoint (RK2) method to obtain a four decimal-place approximation of $y(1.2)$ using

  (i) a step size of 0.1

  (ii) a step size of 0.05

(d) Now use the RK4 method (`ode45()` function in MATLAB) to obtain a four decimal-place approximation of $y(1.2)$ using

  (i) a step size of 0.1

  (ii) a step size of 0.05

Scientifically, the accuracy of your estimated values can be determined by a metric known as mean squared error defined as

$$\text{MSE} = \frac{1}{n+1}\sum_{i=0}^{n}\left(y(t_i) - y_i\right)^2$$

where $y(t_i)$ is an actual value of y at time t_i, and y_i is the estimated value at the i^{th} iteration step.

(e) Evaluate the MSEs for $y(1), y(1.1)$ and $y(1.2)$ in

  (i) b(i).

  (ii) c(i).

  (iii) d(i).

(f) Evaluate the MSEs for $y(1), y(1.05), y(1.1), y(1.15)$ and $y(1.2)$ in

  (i) b(ii).

  (ii) c(ii).

  (iii) d(ii).

(g) Using the insights from parts (e) and (f)

  (i) Explain how decreasing the step size affects the accuracy of the estimated solution?

  (ii) Rank the three methods based on their accuracy. Give appropriate reasoning based on the values of MSE.

Problem 5

Open the following link and fill out the midterm course evaluation survey for this course. This survey is totally anonymous. Your feedback is very valuable to me as this will help me improve this course and teach you better for the rest of the semester. So please try to be as honest as possible with your responses. If you dislike something about the course, you are encouraged to pour your heart out in the comments.

`https://goo.gl/forms/rHHYkPFjSY2T8PT92`

Answer the following questions while filling out the survey and write down the answers on your assignment.

(a) What is the 12^{th} question in the survey?

(b) How many total questions are there in the survey?

Complete the survey and write down on your assignment "I have submitted the survey".