

Roll# Student 1: _____

Roll# Evaluator 1: _____

Roll# Student 2: _____

Roll# Evaluator 2: _____

Note: Attempt the questions in chronological order.

Problem 1 _____ / [10 Marks]

Find the derivative implicitly of the following functions

(a) $\cos(xy) = 1 + \sin(y)$

(b) $e^y \sin(x) = x + xy$

Problem 2 _____ / [10 Marks]

Find the equation of tangent line to the curve $y = f(x)$ at the point $(0, \frac{1}{2})$

$$x^2 + y^2 = (2x^2 + 2y^2 - x)^2$$

Problem 3 _____ / [10 Marks]

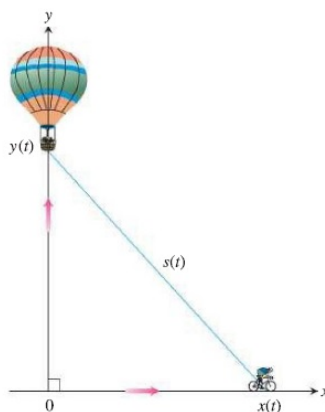
By using the logarithmic differentiation, Find the derivatives of the following functions

(a) $y = \sqrt{\frac{x-1}{x^4+1}}$

(b) $y = x^{\cos(x)}$

Problem 4 _____ / [20 Marks]

A balloon is rising vertically above a level, straight road at a constant rate of 1 ft/sec. Just when the balloon is 65 ft above the ground, a bicycle moving at a constant rate of 17 ft/sec passes under it. How fast is the distance between the bicycle and balloon increasing 3 sec later?



Problem 5 _____ / [10 Marks]

The number of yeast cells in a laboratory culture increases rapidly initially but levels off eventually. The population is modeled by the function

$$n = f(t) = \frac{a}{1 + be^{-0.7t}}$$

where n is measured in hours. At time $t = 0$ the population is 20 cells and is increasing at a rate of 12 cells per hour. Find the values of a and b . According to this model, what happens to the yeast population in the long run?

Problem 6 _____ / [20 Marks]

Two carts, A and B, are connected by a rope 39 ft long that passes over a pulley P (see the figure). The point Q is on the floor 12 ft directly beneath P and between the carts. Cart A is being pulled away from Q at a speed of 2 ft/s. How fast is cart B moving toward Q at the instant when cart A is 5 ft from Q?

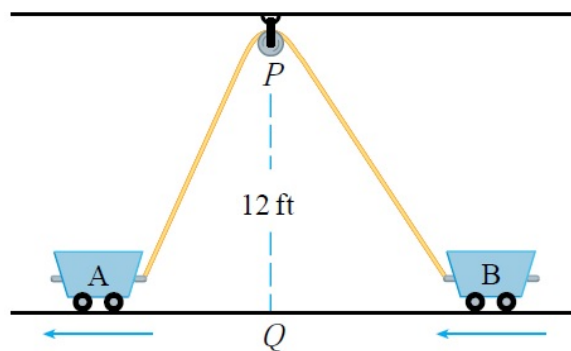


Figure 1: Pile of dump