Roll# Student 1: ---------------------

Roll# Evaluator 1: ---------------------

Roll# Student 2: ---------------------

Roll# Evaluator 2: ---------------------

Problem 1 $[10 = 5 + 5]$

You plugged your mobile phone into a charger. After some time you removed it from charging and started watching Youtube videos on it until it warns you "battery low". The graph shows $C(t)$, the battery charge percentage as a function of time t elapsed (in hours).
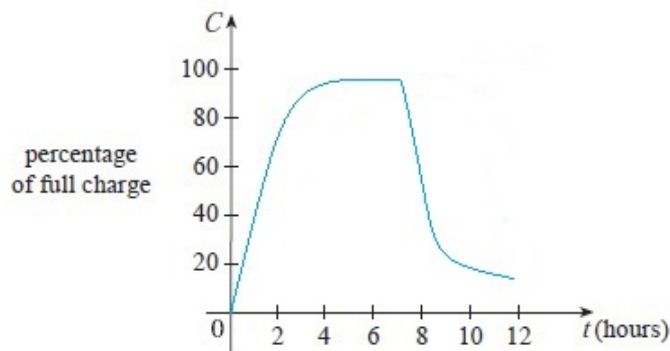
(a) What is the meaning of the derivative $C'(t)$?

(b) Sketch the graph of $C'(t)$. What does the graph tell you about charging and discharging?

Problem 2 $[20 = 5 + 5 + 5 + 5]$

The graph of f is given. State, with reasons, the values of x at which f is not differentiable.
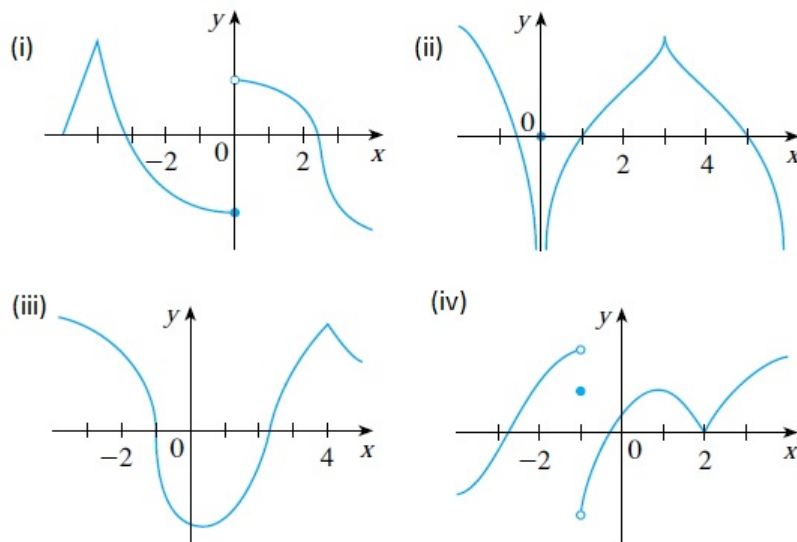
Problem 3 $[10 = 5 + 5]$

The production cost (in dollars) of producing x units of a certain commodity is

$$P(x) = 5 + 10x + 2x^2$$

(a) Find the average rate of change of P with respect to x when the production level is changed

    (i) from $x = 100$ to $x = 105$

    (ii) from $x = 100$ to $x = 101$

(b) Find the instantaneous rate of change of P with respect to x when $x = 100$

Problem 4 $[20 = 10 + 10]$

(a) Determine whether $f'(0)$ exists for

$$f(x) = \begin{cases} x^2 \sin\left(\dfrac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$$

(b) Determine whether $f'(-1)$ exists for $f(x) = |x + 1|$.

Hint: Use the limit definition of derivative $f'(a) = \lim\limits_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$

Problem 5 $[10]$

Find the derivative function of $f(x) = \sin 2x$ using the limit definition of derivative (a.k.a. derivative from first principles). Hint: $\sin A - \sin B = 2 \cos\left(\dfrac{A+B}{2}\right) \sin\left(\dfrac{A-B}{2}\right)$

Problem 6 $[10 = 5 + 5]$

Let

$$f(x) = \begin{cases} x^2 & x \leq 2 \\ mx + c & x > 2 \end{cases}$$

Find the values of m and c that makes f differentiable everywhere

Hint: Continuity is not sufficient for differentiability. It is just necessary.