MT110: Calculus and Analytic Geometry

Worksheet 6

Due 3:15 pm, Fri Oct 12

Fall 2018

Roll# Student 1: _____ Roll# Evaluator 1: _____

Roll# Student 2: _____ Roll# Evaluator 2: _____

Problem 1 [10 marks]

Determine the continuity of the following functions at the points given below.

(a)

$$f(x)\begin{cases} \frac{x-6}{x-3} & x<0 \\ 2 & x=0 \\ \sqrt{4+x^2} & x>0 \end{cases}, \text{ at } x=0$$

Problem 2 [$10 = 2 \times 5$ marks]

Find the numbers at which f is discontinuous. At which of these numbers is f continuous from the right, from the left, or neither? Also write the interval of continuity.

(a)

$$f(x)=\begin{cases} 1+x^2 & x\leq 0 \\ 2-x & 0<x\leq 2 \\ (x-2)^2 & x>2 \end{cases}$$

Problem 3 [5 marks]

For what value of a is $f(x)$ continuous at every x?

$$f(x)=\begin{cases} x^2-1 & x<3 \\ 2ax & x\geq 3 \end{cases}$$

Problem 4 [$15 = 3 \times 5$ marks]

Find the discontinuity of the following functions and classify them, if required then modify the function so that it becomes continuous.Also, find the interval of continuity.
Note: Functions are defined on the real domain $\mathbb{R}$

(a) $f(x)=\dfrac{\sin x}{x}$

(b) $f(t)=\dfrac{t^2+3t-10}{t-2}$

(c) $f(x)=\dfrac{x^2-16}{x^2-3x-4}$

(d) $f(x)=\dfrac{x+2}{\cos x}$

(e) $f(x)=\dfrac{x\tan x}{x^2+1}$

Problem 5 [10 marks]

If $f(x) = x^3 - 8x + 10$, show that there are values $c \in [a, b] \subset \mathbb{R}$ for which $f(c) = \pi$.

Problem 6 [10=5+5 marks]

(a) How close to -4 do we have to take x so that $\dfrac{1}{(x+4)^2} > 10000$

(b) Using $\epsilon - \delta$ definition to show that $\lim\limits_{x \to -4} \left(\dfrac{1}{(x+4)^2} \right) = \infty$

Problem 7 [$10 = 4 \times 2.5$ marks]

Evaluate the following limit

(a) $\lim\limits_{x \to \infty} \left(\dfrac{\sin x}{x} \right)$

(b) $\lim\limits_{x \to \infty} \left(e^{-x} + \dfrac{2 \cos x}{x} \right)$

(c) $\lim\limits_{x \to -\infty} \left(\dfrac{1 - e^{-x}}{1 + 2e^{-x}} \right)$

(d) $\lim\limits_{x \to -\infty} \left(\dfrac{2x^2}{\sqrt{4x^4 + 1}} \right)$