

Roll# Student 1:

Roll# Evaluator 1:

Roll# Student 2:

Roll# Evaluator 2:

Problem 1 [10 marks]

Sketch the graph of the function f and use it to determine the values of $\lim_{x \rightarrow 1} f(x)$ and $\lim_{x \rightarrow -1} f(x)$.

$$f(x) = \begin{cases} 1 + x & \text{if } x < -1 \\ x^2 & \text{if } -1 \leq x < 1 \\ 2 - x & \text{if } x \geq 1 \end{cases}$$

Problem 2 [5 marks]

A patient receives a 150-mg injection of a drug every 4 hours. The graph shows the amount of $f(t)$ the drug in the blood stream after t hours. Find $\lim_{t \rightarrow 12^-} f(t)$ and $\lim_{t \rightarrow 12^+} f(t)$ and explain the significance of these one-sided limits.

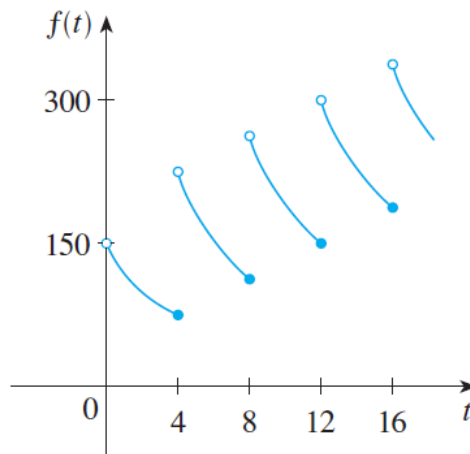


Figure 1: Graph 1

Problem 3 [5 marks]

The signum (or sign) function, denoted by sgn , is defined by

$$\text{sgn}(x) = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$$

a) Sketch the graph of this function. [2]

b) Find each of the following limits or explain why it does not exist.

i) $\lim_{x \rightarrow 0^+} \operatorname{sgn}(x)$ [1]

iii) $\lim_{x \rightarrow 0} \operatorname{sgn}(x)$ [1]

ii) $\lim_{x \rightarrow 0^-} \operatorname{sgn}(x)$ [1]

Problem 4 [24 marks]

Evaluate the limit and justify each step by indicating the appropriate Limit Law(s).

a) $\lim_{x \rightarrow 3} (5x^3 - 3x^2 + x - 6)$ [2]

g) $\lim_{x \rightarrow 2} \sqrt{\frac{2x^2 + 1}{3x - 2}}$ [3]

b) $\lim_{x \rightarrow -1} (x^4 - 3x)(x^2 + 5x + 3)$ [2]

c) $\lim_{t \rightarrow -2} \frac{t^4 - 2}{2t^2 - 3t + 2}$ [2]

h) $\lim_{t \rightarrow 1} \frac{t^4 - 1}{t^3 - 1}$ [3]

d) $\lim_{u \rightarrow -2} \sqrt{u^4 + 3u + 6}$ [2]

i) $\lim_{x \rightarrow 4} \frac{x^2 - 4x}{x^2 - 3x - 4}$ [3]

e) $\lim_{x \rightarrow 8} (1 + \sqrt{x})(2 - 6x^2 + x^3)$ [2]

f) $\lim_{t \rightarrow 2} \left(\frac{t^2 - 2}{t^3 - 3t + 5} \right)^2$ [2]

j) $\lim_{t \rightarrow 0} \left(\frac{1}{t} - \frac{1}{t^2 + t} \right)$ [3]

Problem 5 [16 marks]

Determine whether the following limit is positive infinity, negative infinity or undefined.

(a) $\lim_{x \rightarrow -3} \frac{x + 2}{x + 3}$ [4]

(c) $\lim_{x \rightarrow 3} \ln(x^2 - 9)$ [4]

(b) $\lim_{x \rightarrow 1} \frac{2 - x}{(x - 1)^2}$ [4]

(d) $\lim_{x \rightarrow 2} \frac{x^2 - 2x}{x^2 - 4x + 4}$ [4]

Problem 6 [10 marks]

Show that $\lim_{x \rightarrow 0} x^2 \cos\left(\frac{20\pi}{x}\right) = 0$. Illustrate by graphing the functions $f(x) = -x^2$, $g(x) = x^2 \cos\left(\frac{20\pi}{x}\right)$, and $h(x) = x^2$ on the same screen.

Problem 7 [10 marks]

Use ϵ - δ definition of limit to show that $\lim_{x \rightarrow 6} \left(1 - \frac{x}{3}\right) = -1$.
