MT110: Calculus and Analytic Geometry
Worksheet 13

Roll\# Student 1: $\qquad$ Roll\# Evaluator 1: $\qquad$
Roll\# Student 2: $\qquad$ Roll\# Evaluator 2: $\qquad$
Note: Attempt the questions in a proper sequence.

## Problem 1

 /[10 Marks]The graph of $g$ (Fig:1)consists of two straight lines and a semi -circle. Use it to evaluate the integral.

$$
I=\int_{0}^{7} g(x) d x
$$



Figure 1: $g(x)$

## Problem 2 /[10 Marks]

(i) Express the limit as a definite integral on the given interval.

$$
I_{1}=\lim _{x \rightarrow \infty} \sum_{i=1}^{n} \frac{x_{i}}{x_{i}^{2}+4} \Delta x, \quad[1,3]
$$

(ii) Express the a definite integral on the given interval as a Riemann sum.

$$
I_{2}=\int_{0}^{2} \frac{1}{x^{2}+2} d x
$$

(a) Use the property 8 (page 381) to estimate the integral $\int_{0}^{2} \frac{1}{1+x^{2}} d x$
(b) For the following integral

$$
\int_{0}^{6} \frac{x}{x+1} d x
$$

(i) Use property 8 to estimate its bounds
(ii) Use the Midpoint Rule with $n=3$ to approximate the integral.

## Problem 4 /[20 Marks]

(i) Use the FOTC( Fundamental theorem of calculus) part 1 to find the derivative.

$$
\frac{d}{d x} \int_{\sin (x)}^{1} \sqrt{1+t^{2}} d t
$$

(ii) Use the FOTC( Fundamental theorem of calculus) part 2 to find the integrals.
(a) $I_{4}=\int_{0}^{3}\left(2 \sin (x)-e^{x}\right) d x$
(b) $I_{5}=\int_{-2}^{2} f(x) d x \quad$ where
$f(x)= \begin{cases}2, & -2 \leq x \leq 0 \\ 4-x^{2}, & 0<x \leq 2\end{cases}$

## Problem 5 <br> $\qquad$ /[10 Marks]

A honeybee population starts with 100 bees and increases at a rate of $n^{\prime}(t)$ bees per week. What does $100+\int_{0}^{15} n^{\prime}(t) d t$ represents?

## Problem 6 _ /[10 Marks]

If $f(x)$ is the slope of a trail at a distance of $x$ miles from the start of the trail, what does $\int_{3}^{5} f(x) d x$ represents?

## Problem 7 _ /[10 Marks]

Water flows from the bottom of a storage tank at a rate of $r(t)=400-4 t$ liters per minute, where $0 \leq t \leq 50$. Find the amount of water that flows out from the tank between 10 min to 20 min ?

## Problem 8

Turn off your PCs and place the chairs in their proper positions.

