MT110: Calculus and Analytic Geometry

Homework 6

Due: Fri, Dec 21, 2:00 pm

Fall 2018

Tips to avoid plagiarism

- Solve all parts of a problem in the proper sequence.
- Do not copy the solutions of your classmates.
- Your are encouraged to discuss the problems with your classmates in whatever way you like but make sure to REPRODUCE YOUR OWN SOLUTIONS in what you submit for grading.
- Cite all the online sources that you get help from.
- Keep your work in a secure place.

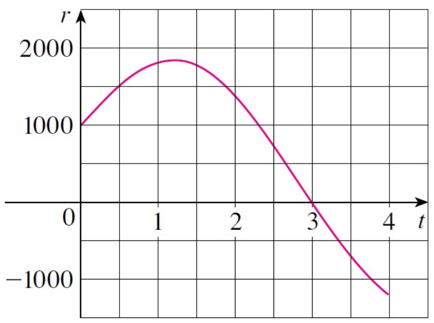
Problem 1

Water flows into and out of a storage tank. A graph of the rate of change $r(t)$ of the volume of water in the tank, in liters per day, is shown in the following. If the amount of water in the tank at time $t = 0$ is $25,000$ liters.

(a) Using the trend in the graph of $r(t)$, describe how the amount of water in the tank is changing from: $t = 0$ to 1.25, $t = 1.25$ to 3, and $t = 3$ to 4. Be careful with the fact that the graph is not of the amount of water. It is for the rate of change of the amount of water.

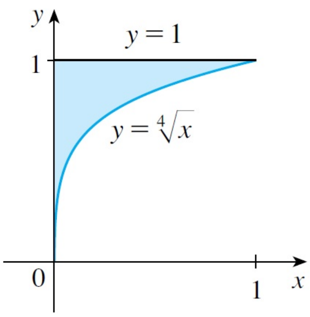
(b) Estimate the amount of water in the tank four days later.

(c) Find an upper bound of the error in your estimate.



Problem 2

Find the exact value of the shaded area shown in the following figure.



Problem 3

If $f(x)$ is continuous on $[a, b]$,

(a) Show that

$$\left|\int_a^b f(x)dx\right| \leq \int_a^b |f(x)|dx$$

Hint: $-|f(x)| \leq f(x) \leq |f(x)|$

(b) Use the part (a) to show that

$$\left|\int_0^{2\pi} f(x)\sin(2x)dx\right| \leq \int_0^{2\pi} |f(x)|dx$$

Problem 4

Use the Fundamental Theorem of Calculus part 1 to find the following derivatives.

(a)

$$\frac{d}{dx}\int_0^{\tan x} \sqrt{t+\sqrt{t}}dt$$

(b)

$$\frac{d}{dx}\int_{1-3x}^{x^2} \frac{u^3}{1+u^2}du$$

Problem 5

For each of the following definite integrals,

(a) $\int_0^{\pi} \sin x \sec^2(\cos x)dx$

(b) $\int_0^{\pi} e^{\cos x}\sin 2x \ dx$

(i) Find the bounds of the integral using an appropriate property.

(ii) First evaluate the indefinite form of the integral using an appropriate substitution.

(iii) Now evaluate the definite integral.

(iv) Using your answer to (iii), evaluate the integral from $-\pi$ to π.

(v) Express the integral as an integral function $\int_a^x f(t)dt$, where a and $f(t)$ are to be specified.

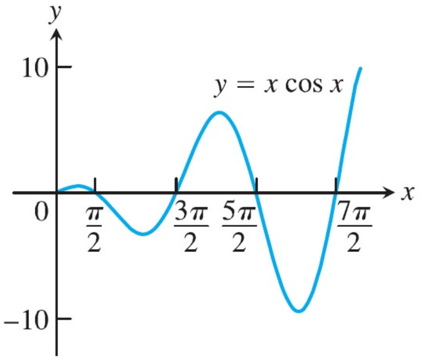
Problem 6

(a) Find the area of the region enclosed by the curve $y = x\cos x$ and the x-axis (see figure) for

(i) $\frac{\pi}{2} \le x \le \frac{3\pi}{2}$

(ii) $\frac{3\pi}{2} \le x \le \frac{5\pi}{2}$

(iii) $\frac{5\pi}{2} \le x \le \frac{7\pi}{2}$



(b) What pattern do you see, i.e. what is the area between the curve and the x-axis for

$$\left(\frac{2n-1}{2}\pi\right) \le x \le \left(\frac{2n+1}{2}\pi\right)$$

where n is an arbitrary positive integer? Give reasons for your answer.

Problem 7

Using the concept of Strategy for Integration "Section 7.5", evaluate the following integrals.

(i) $\displaystyle\int r^2 \left(\frac{r^3}{18} - 1\right)^5 dr$

(ii) $\displaystyle\int \sqrt{\frac{x^4}{x^3 - 1}}\, dx$

(iii) $\displaystyle\int \frac{\sec t \tan t}{\sqrt{\sec t}} dt$

(iv) $\displaystyle\int e^x \sin(e^x) dx$

(v) $\displaystyle\int \sec^6 \theta d\theta$

(vi) $\displaystyle\int \frac{\sin^3 x}{\cos^4 x} dx$

(vii) $\displaystyle\int \frac{dx}{4 + x^2}$

(viii) $\displaystyle\int \frac{x^3}{x^2 - 1} dx$

(ix) $\displaystyle\int \frac{2x^4}{x^3 - x^2 + x - 1} dx$

(x) $\displaystyle\int \frac{\sin(x)}{\cos^2(x) + \cos(x) - 2} dx$

(xi) $\displaystyle\int_{-1}^{2} |e^x - 1| dx$

(xii) $\displaystyle\int_{\pi/4}^{\pi/3} \frac{\ln(\tan(x))}{\sin(x)\cos(x)} dx$

Problem 8

Determine whether each integral is convergent or divergent. Evaluate those that are convergent

(i) $\displaystyle\int_{-\infty}^{\infty} x e^{-2x^2} dx$

(ii) $\displaystyle\int_{1}^{\infty} \frac{e^{-\sqrt{x}}}{\sqrt{x}} dx$

(iii) $\displaystyle\int_{-\infty}^{\infty} \cos(\pi x) dx$

(iv) $\displaystyle\int_{-\infty}^{\infty} \frac{x^2}{4 + x^6} dx$ Hint: use $t = x^3$

(v) $\displaystyle\int_{0}^{\infty} \frac{e^x}{e^{2x} + 3} dx$

(vi) $\displaystyle\int_{-1}^{1} \frac{e^{\frac{1}{x}}}{x^3} dx$

(vii) $\displaystyle\int_{6}^{8} \frac{4}{(x - 6)^3} dx$
