MT110: Calculus and Analytic Geometry

Homework 5

Due: Fri, Dec 7, 2:00 PM (no extension)

Fall 2018

Tips to avoid plagiarism

- Do not copy the solutions of your classmates.
- Your are encouraged to discuss the problems with your classmates in whatever way you like but make sure to REPRODUCE YOUR OWN SOLUTIONS in what you submit for grading.
- Cite all the online sources that you get help from.
- Keep your work in a secure place.

Problem 1

Evaluate the following limits.

(a) $\lim\limits_{x\to -2} \dfrac{x+2}{\ln(x+3)}$

(b) $\lim\limits_{x\to \infty} \dfrac{\ln x}{\sqrt{x}}$

(c) $\lim\limits_{x\to \infty} x\tan\frac{1}{x}$

(d) $\lim\limits_{x\to 0^+} x^{-\sqrt{x}}$

(e) $\lim\limits_{x\to 0^+} \dfrac{\ln x}{x}$

(f) $\lim\limits_{x\to 1^+} x^{\frac{2}{1-x}}$

Problem 2

Read the curve sketching guidelines on page 311 in section 4.5 (Stewart) to sketch the following curves by hand. Mention all the steps. Note: Reading the examples in that section will be very helpful.

(a) $y = x(x-4)^3$

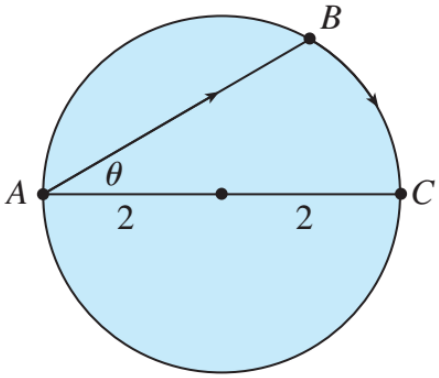
(b) $y = (1-x)e^x$

(c) $y = \dfrac{x}{x^3-1}$

(d) $y = \dfrac{\sin x}{2+\cos x}$

Problem 3

A woman at a point A on the shore of a circular lake with radius 2 km wants to arrive at the point C diametrically opposite to A on the other side of the lake. She can walk at the rate of 4 km/h and row a boat at 2 km/h. Find the path that she must take in order to go from A to C in the shortest possible time?



Problem 4

A piece of wire 10 m long is cut into two pieces. One piece is bent into a square and the other piece is bent into a circle. What is the length of each piece so that the total area enclosed in the two shapes is

(a) a maximum?

(b) a minimum?

(c) Evaluate the maximum and minimum possible enclosed areas.

Problem 5

For a fish swimming at a speed v relative to the water, the energy expenditure per unit time is proportional to v^3. It is believed that migrating fish try to minimize the total energy required to swim a fixed distance. If the fish are swimming against an ocean current u (for $u < v$), then the time required to swim a distance L is $\frac{L}{v-u}$ and the total energy E required to swim the distance is given by

$$E(v) = av^3 \frac{L}{v-u},$$

where a is the constant of proportionality.

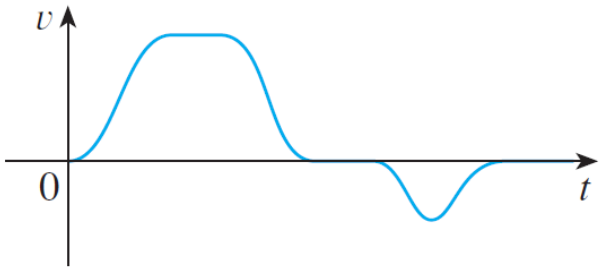
(a) Determine the value of v that minimizes E.

(b) Sketch the graph of E.

(c) It has been experimentally found that the migrating fish swim against an ocean current at a speed that is 50% greater than the current speed. How accurate is the model equation $E(v)$?

Problems 6

The graph of the velocity function of a particle is shown in the figure. Sketch the graph of a position function.



Problems 7

A particle is moving with the given data. Find the position of the particle.

$$a(t) = 3\cos t - 2\sin t,\ s(0) = 0,\ v(0) = 4$$

Problems 8

(a) Use six rectangles to find estimates of each type for the area under the given graph of f from $x = 0$ to $x = 12$.

(i) L_6 (sample points are left endpoints)

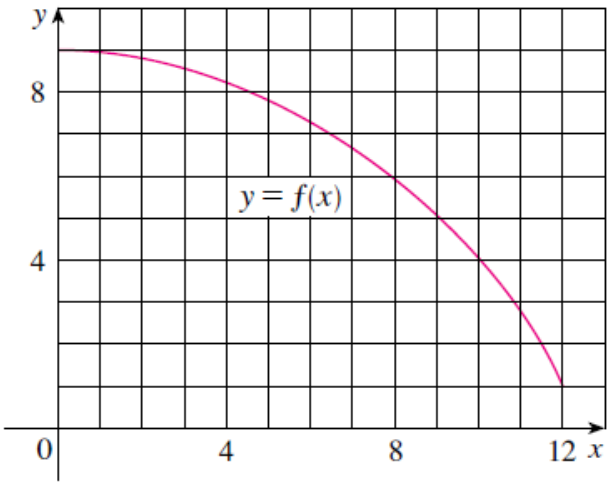
(ii) R_6 (sample points are right endpoints)

(iii) M_6 (sample points are midpoints)

(b) Is L_6 an underestimate or overestimate of the true area?

(c) Is R_6 an underestimate or overestimate of the true area?

(d) Which of the numbers L_6, R_6, M_6 or gives the best estimate? Explain.



Problem 9

Speedometer readings for a motorcycle at 12-second intervals are given in the table.

(a) Estimate the distance traveled by the motorcycle during this time period using the velocities at the beginning of the time intervals.

(b) Give another estimate using the velocities at the end of the time periods.

(c) Are your estimates in parts (a) and (b) upper and lower estimates? Explain.

$t(s)$	0	12	24	36	48	60
$v(ft/s)$	30	28	25	22	24	27

Problem 10

Verify that R_n and L_n gives the same value as $\lim_{n\to\infty}$ for the given integral

$$\int_1^4 (x^2 - 4x + 2)\, dx,$$