MT110: Calculus and Analytic Geometry

Homework 4

Due: Fri, Nov 23, 2:00 pm

Fall 2018

Tips to avoid plagiarism

- Do all the parts of a problem in a proper sequence.
- Do not copy the solutions of your classmates.
- Your are encouraged to discuss the problems with your classmates in whatever way you like but make sure to REPRODUCE YOUR OWN SOLUTIONS in what you submit for grading.
- Cite all the online sources that you get help from.
- Keep your work in a secure place.

Problem 1

Find the derivative of the following functions using chain rule.

(a) $y = \left(\frac{x^2}{8} + x - \frac{1}{x}\right)^4$

(b) $y = \cot\left(\pi - \frac{1}{x}\right)$

(c) $y = (\csc(x) + \cot(x))^{-1}$

(d) $y = 4\sin\left(\sqrt{1+\sqrt{t}}\right)$

Problem 2

Find the derivative of the following functions using implicit differentiation or use the logarithmic differentiation where applicable.

(a) $y\sin\left(\frac{1}{y}\right) = 1 - xy$

(b) $y^2\cos\left(\frac{1}{y}\right) = 2x + 2y$

(c) $x^{\ln(x)}$

(d) $x^y = y^x$

Problem 3

Find the equation of the line tangent to the curve $xy - 2x - y = 0$ at $x = 4$

Problem 4

Gravel is being dumped from a conveyor belt at a rate of 30 ft^3/min as shown in Fig. 1. It forms a pile in the shape of a right circular cone whose base diameter and height are always the same. How fast is the height of the pile increasing when the pile is 17 ft high?

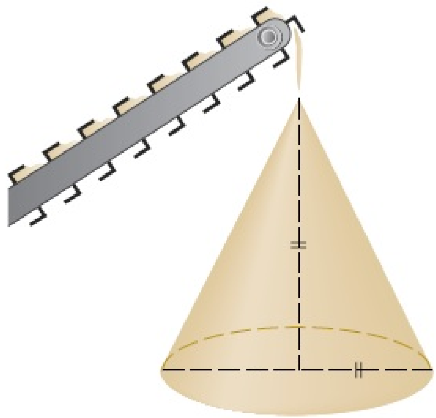
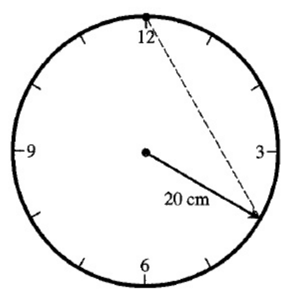


Figure 1: Pile of dump

Problem 5

At what rate is the distance between the tip of second hand and the 12 o'clock mark changing when the second hand points to 4 o'clock?



Problem 6

In a fish farm, a population of fish is introduced into a pond and harvested regularly. A model for the rate of change of the fish population is given by the equation

$$\frac{dP(t)}{dt} = r_0 \left(1 - \frac{P(t)}{P_c}\right) P(t) - \beta P(t)$$

where r_0 is the birth rate of the fish, P_c is the maximum population that the pond can sustain (called the carrying capacity), and β is the percentage of the population that is harvested.

(a) What $\frac{dP}{dt}$ value of corresponds to a stable population?

(b) If the pond can sustain 10,000 fish, the birth rate is 5%, and the harvesting rate is 4%, find the stable population level.

(c) What happens if β is raised to 5%?

Problem 7

Find the absolute extrema of the following functions. State the possible critical points in the given interval.

[Note: You don't need to look anywhere online about how to differentiate the absolute value functions and apply all those strange methods. It cannot be emphasized enough that drawing a graph is always helpful. So to find their critical points, just draw their graphs.]

(a) $f(x) = \frac{2}{3}x^2 - 5 \quad -2 \leq x \leq 3$

(b) $f(x) = \sqrt[3]{x} \quad -1 \leq x \leq 8$

(c) $f(x) = 2 - |x| \quad -1 \leq x \leq 3$

(d) $f(x) = |x - 5| \quad 4 \leq x \leq 7$

(e) $f(x) = |\cos(2x)| \quad 0 \leq x \leq \pi$

Problem 8

For the following functions

(a) $g(t) = -3t^2 + 9t + 5$

(b) $f(x) = \frac{x^4}{4} - x^3 - 3x^2 + 8x$

(c) $\dfrac{x^3}{3x^2 + 1}$

(d) $x^{2/3}(x^2 - 4)$

(i) Using the first derivative function test for extremum, find the interval where the derivative is increasing or decreasing then decide about the local extremum of the functions and classify them.

(ii) Sketch the graph of the functions using above information.

Problem 9

Show that the equation $x = \cos(x)$ has exactly one solution.

Problem 10

The height above ground of object moving vertically is given as

$$s = -16t^2 + 96t + 112$$

with s in feet and t is in seconds. Find

(a) the object's velocity when $t = 0$

(b) its maximum height and when it occurs

(c) its velocity when $s = 0$
