Tips to avoid any cases of plagiarism

- Do not look at the solutions of your classmates.

- Your are encouraged to discuss the homework with your classmates, but restrict yourself to oral discussions only.

- Cite all the online sources that you get help from.

- Keep your work in a secure place.

Problem 1

(a) Let $f(x) = \dfrac{x-1}{\sqrt{x}-1}$

    (i) Evaluate $f(1.5), f(1.1), f(1.01)$ and $f(1.001)$ and guess the value for $\lim\limits_{x \to 1^+} f(x)$.

    (ii) Evaluate $f(0.5), f(0.9), f(0.99)$ and $f(0.999)$ and guess the value for $\lim\limits_{x \to 1^-} f(x)$.

    (iii) Do you think $\lim\limits_{x \to 1} f(x)$ exists?

(b) Let $f(x) = \dfrac{x+1}{x^2-1}$

    (i) Evaluate $f(-1.5), f(-1.1), f(-1.01)$ and $f(-1.001)$ and guess the value for $\lim\limits_{x \to -1^-} f(x)$.

    (ii) Evaluate $f(-0.5), f(-0.9), f(-0.99)$ and $f(-0.999)$ and guess the value for $\lim\limits_{x \to -1^+} f(x)$.

    (iii) Do you think $\lim\limits_{x \to -1} f(x)$ exists?

Problem 2

Sketch a possible graph of each of the following functions with the given properties.

(a) $f(-1) = 2, f(0) = -1, f(1) = 3$ and $\lim\limits_{x \to 1} f(x)$ does not exist.

(b) $f(x) = 1$ for $-2 \le x \le 1$, $\lim\limits_{x \to 1^+} f(x) = 3$ and $\lim\limits_{x \to -2} f(x) = 1$.

(c) $f(0) = 1$, $\lim\limits_{x \to 0^-} f(x) = 2$ and $\lim\limits_{x \to 0^+} f(x) = 3$.

(d) $\lim\limits_{x \to 0} f(x) = -2$, $f(0) = 1$, $f(2) = 3$ and $\lim\limits_{x \to 2} f(x)$ does not exist.

Problem 3

(a) Numerically estimate $\lim\limits_{x \to 0^+} x^{\sec x}$

(b) Explain what is wrong with the following logic:

$$\text{Since 0 to any power is 0, } \lim_{x \to 0} x^{\sec x} = \lim_{x \to 0} 0^{\sec x} = 0.$$

Problem 4

(a) Give a possible expression of a function $f(x)$ such that $\lim\limits_{x \to 0} f(x)$ exists but $f(0)$ does not exist.

(b) Give a possible expression of a function $g(x)$ such that $g(0)$ exists but $\lim\limits_{x \to 0} g(x)$ does not exist.

(c) Give a possible expression of a function $f(x)$ such that $f(0)$ exists and $\lim\limits_{x \to 0} f(x)$ exists but $\lim\limits_{x \to 0} f(x) \neq f(0)$.

Problem 5

Find the exact value of the following limits. State clearly any limit rules or theorems that you use.

(a) $\lim\limits_{x \to 2} \dfrac{x+2}{x^2 + 5x + 6}$

(b) $\lim\limits_{x \to -1} \dfrac{x^2 - 1}{x + 1}$

(c) $\lim\limits_{x \to 1} \dfrac{x - 1}{\sqrt{x + 3} - 2}$ [Hint: Rationalize]

(d) $\lim\limits_{x \to 1} \dfrac{x^2 - 1}{x + 2}$

(e) $\lim\limits_{x \to -1} \dfrac{x^2 + x}{x^2 - x - 2}$

(f) $\lim\limits_{x \to 2} \dfrac{x^2 - 2x}{x^2 - 4}$

(g) $\lim\limits_{x \to \pi} x \sin x$

(h) $\lim\limits_{x \to 0} \dfrac{\tan x}{\sin x}$

(i) $\lim\limits_{x \to 2} \sqrt{x^2 - 2x + 2}$

(j) $\lim\limits_{x \to 0} \dfrac{1 - \cos x}{x^2}$

(k) $\lim\limits_{x \to 0} x^4 \cos \left(\dfrac{2}{x} \right)$

Problem 6

(a) Write each answer as an equality, and any decimals up to 7 decimal places (e.g. $2 < x < 2.0164389$). Clearly state the value of ϵ and δ (or M or N and δ) in each case.

   (i) How close to 4 do we need to take x so that $\left(\frac{x}{2} - 2 \right) < 0.001$?

   (ii) How close to 4 do we need to take x so that $\left(\frac{x}{2} - 2 \right) > -0.0001$?

   (iii) How close to 0 do we need to take x so that $(2x + 9) < 9.0001$?

   (iv) How close to 0 do we need to take x so that $(2x + 9) > 8.999$?

   (v) How close to 0 do we need to take x so that $\left(x^2 + 6x + 9 \right) < 9.001$?

   (vi) How close to 0 do we need to take x so that $\left(x^2 + 6x + 9 \right) < 9.0001$?

   (vii) How close to 0 do we need to take x so that $\left(x^2 + 6x + 9 \right) > 8.9999$?

   (viii) How close to -7 do we need to take x so that $\frac{1}{(x+7)^4} > 10000$?

   (ix) How close to -7 do we need to take x so that $\frac{1}{(x+7)^4} > 100000$?

   (x) How close to 0 do we need to take x so that $\ln x < -10000$?

   (xi) How close to 0 do we need to take x so that $\ln x < -100000$?

(b) Use the ϵ-δ definition of a limit to show that

   (i) $\lim\limits_{x \to 4} \left(\dfrac{x}{2} - 2 \right) = 0$

   (ii) $\lim\limits_{x \to 0} (2x + 9) = 9$

   (iii) $\lim\limits_{x \to 0} (x^2 + 6x + 9) = 9$

   (iv) $\lim\limits_{x \to -7} \dfrac{1}{(x + 7)^4} = \infty$

   (v) $\lim\limits_{x \to 0^+} \ln x = -\infty$