MT110: Calculus and Analytic Geometry

Homework 1

Due 2:00 pm, Friday Sep 28

Fall 2018

Tips to avoid any cases of plagiarism

- You must NOT look at the solutions of your classmates.
- Your are encouraged to discuss the homework with your classmates, but restrict yourself to oral discussions only.
- Cite all the online sources that you get help from.
- Keep your work in a secure place.

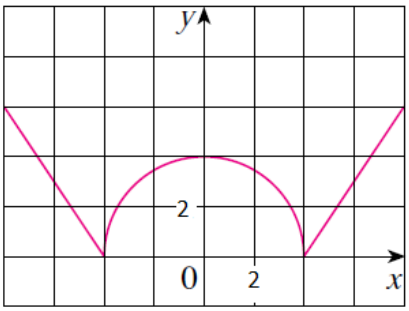
Problem 1

Draw the following regions in Cartesian plane.

(a) $R = \{x, y : -7 \leq x < 2 \text{ and } -10 < y < 5\}$ (b) $R = \{x, y : 2x^2 + 2y^2 < 32\}$

Problem 2

Find an expression for the function whose graph is the following curve. [Hint: Define the function piecewise for three different intervals in its domain]



Problem 3

The manager of a weekend flea market knows from past experience that if he charges dollars for a rental space at the market, then the number of spaces he can rent is given by the equation $y = 250 - 3x$.

(a) Sketch a graph of this linear function. (Remember that the rental charge per space and the number of spaces rented cannot be negative quantities.)

(b) What do the slope, the x-intercept, and the y-intercept of the graph represent in the actual scenario?

Problem 4

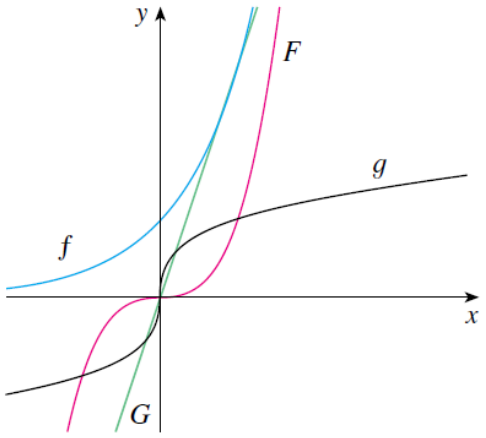
Match each of the equations (a), (b), (c) and (d) with its corresponding graph marked by f, F, g and G in the following figure. Explain your choice.

(a) $y = 3x$

(b) $y = 3^x$

(c) $y = x^3$

(d) $y = \sqrt[3]{x}$



Problem 5

Graph the function by hand, not by plotting points, but by starting with the graph of one of the standard functions and then applying the appropriate transformations one step at a time.

(a) $y = \dfrac{1}{x+2}$

(b) $y = (x-1)^3$

(c) $y = -\sqrt[3]{x}$

(d) $y = x^2 + 6x + 4$

(e) $y = \sqrt{x-2} - 1$

(f) $y = 4\sin 3x$

(g) $y = \sin \frac{1}{2}x$

(h) $y = \dfrac{2}{x} - 2$

(i) $y = \dfrac{1}{2}(1 - \cos x)$

(j) $y = 1 - 2\sqrt{x+3}$

(k) $y = 1 - 2x - x^2$

(l) $y = \mid x \mid -2$

(m) $y = \mid x - 2 \mid$

(n) $y = \dfrac{1}{4}\tan\left(x - \dfrac{\pi}{4}\right)$

Problem 6

Given $f(x) = \sqrt{3-x}$ and $g(x) = \sqrt{x^2-1}$. Find the following combinations of functions and state the domain in each case.

(a) $f + g$ (b) $f - g$ (c) fg (d) f/g

Problem 7

Let $f(x) = x + \dfrac{1}{x}$ and $g(x) = \dfrac{x+1}{x+2}$, find the following composite functions and their domains.

(a) $f \circ g$

(b) $g \circ f$

(c) $f \circ f$

(d) $g \circ g$

Problem 8

(a) Suppose f is a one-to-one function with domain D and range E. How is the inverse function f^{-1} defined? What is the domain of f^{-1}? What is the range of f^{-1}?

(b) If you are given a formula for f, how do you find a formula for f^{-1}?

(c) If you are given the graph of f, how do you find the graph of f^{-1}?

Problem 9

Let $f(x) = \sqrt{1-x^2}, 0 \leq x \leq 1$.

(a) Find $f^{-1}(x)$. How is it related to $f(x)$?

(b) Sketch the graph of f and explain your answer to part (a).

Problem 10

We know that domain of function $f(x) = x^2$ is all real numbers because we can square all real numbers. Why is it true that its range consists of all positive real numbers, without leaving out any numbers in between? [Hint: Use the concept of an inverse function]

Problem 11

For each of the following functions,

(a) $f(x) = \dfrac{4x-1}{2x+3}$

(b) $f(x) = \ln(x+3)$

(i) Find its domain and range

(ii) Find a formula for the inverse of the function and find its domain and range.
