Problem 1

$$I(s) = \left(\frac{1}{R} + \frac{1}{Ls} + Cs\right) V(s)$$

$$I(s) = \left(\frac{Ls + R + RLCs^2}{RLs}\right) V(s)$$

$$\frac{V(s)}{I(s)} = \frac{RLs}{Ls + R + RLCs^2}$$

$$\frac{V(s)}{I(s)} = \frac{\frac{1}{Cs}}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}$$

Problem 2

For writing the differential equation for the displacement x_1 of mass m_1 apply the super position principle, m_1 moving m_2 stationary

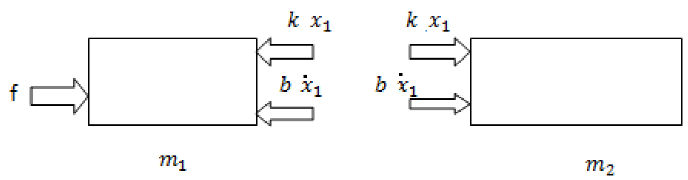

Figure 1: Free Body Diagram of Spring-Mass-Damper System m_1 moving m_2 stationary

Using free body diagram, equation of motion form_1 according to Newton's second law is

$$F_{net} = m_1 a = f - (x_1 - x_2)k - (\dot{x_1} - \dot{x_2})b$$

$$(m_1 s^2 + bs + k)X_1(s) + (-bs - k)X_2(s) = F(s) \tag{2.1}$$

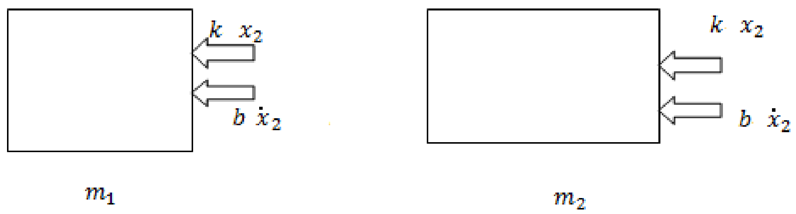

Figure 2: Free Body Diagram of Spring-Mass-Damper System m_2 moving m_1 stationary

For writing the differential equation for the displacement x_2 of mass m_2 apply the super position principle, m_2 moving m_1 stationary

$$m_2\ddot{x_2} = b\dot{x_1} + kx_1 - b\dot{x_2} - kx_2$$

$$m_2\ddot{x_2} + b(\dot{x_2} - \dot{x_1}) + k(x_2 - x_1) = 0$$

$$(-bs - k)X_1(s) + (m_2s^2 + bs + k)X_2(s) = 0 \tag{2.2}$$

Eq(2.1)$\times(-bs-k)$-Eq(2.2)$\times(m_1s^2+bs+k)$

$$(bs+k)^2X_2(s) - (m_1s^2+bs+k)(m_2s^2+bs+k)X_2(s) = F(s)(-bs-k)$$

$$\begin{aligned} X_2(s) &= \frac{bs+k}{(m_1s^2+bs+k)(m_2s^2+bs+k)-(bs+k)^2} \\ &= \frac{bs+k}{m_1m_2s^4+(m_1+m_2)bs^3+(m_1+m_2)ks^2} \end{aligned}$$

$$\frac{X_2(s)}{F(s)} = \frac{\begin{vmatrix} m_1s^2+k+bs & F(s) \\ -(k+bs) & 0 \end{vmatrix}}{\begin{vmatrix} m_1s^2+k+bs & -(k+bs) \\ -(k+bs) & m_2s^2+k+bs \end{vmatrix}}$$

$$\frac{X_2(s)}{F(s)} = \frac{bs+k}{(m_1s^2+bs+k)(m_2s^2+bs+k)-(bs+k)^2}$$

Problem 3

$$\frac{dy}{dt} + 10(1-y) = 2x$$

$$\frac{dy}{dt} - 10y + 10 = 2x$$

Let $u = 2x - 10$

$$\frac{T(s)}{U(s)} = \frac{1}{s-10}$$

Problem 4

$$x = \begin{bmatrix} \dot{y} \\ \theta \\ \dot{\theta} \end{bmatrix}$$

$$\begin{bmatrix} \ddot{y} \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} 0 & \frac{-mg}{M} & 0 \\ 0 & 0 & 1 \\ 0 & \frac{g}{l} & 0 \end{bmatrix} \begin{bmatrix} \dot{y} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} \frac{1}{M} \\ 0 \\ \frac{-1}{Ml} \end{bmatrix} u$$

$$y = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{y} \\ \theta \\ \dot{\theta} \end{bmatrix}$$

Problem 5

$$\frac{Y(s)}{U(s)} = C(sI - A)^{-1}B + D$$

$$(sI - A)^{-1} = \begin{bmatrix} s & 1 \\ -1 & s \end{bmatrix}^{-1} = \frac{1}{s^2+1}\begin{bmatrix} s & -1 \\ 1 & s \end{bmatrix}$$

$$\frac{Y(s)}{U(s)} = \frac{1}{s^2+1}\begin{bmatrix} -1 & 0 \end{bmatrix}\begin{bmatrix} s & -1 \\ 1 & s \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} + 1$$

$$= \frac{1}{s^2+1}\begin{bmatrix} -1 & 0 \end{bmatrix}\begin{bmatrix} s+1 \\ 1-s \end{bmatrix} + 1$$

$$= \frac{s(s+1)}{s^2+1}$$

Problem 6

$$Y(s) = G(s)\frac{1}{s} = \frac{s-1}{s(s+1)(s+2)}$$

$$= \frac{\frac{-1}{2}}{s} + \frac{2}{s+1} + \frac{\frac{-3}{2}}{s+2}$$

$$y(t) = \frac{-1}{2} + 2e^{-t} - \frac{3}{2}e^{-2t}$$

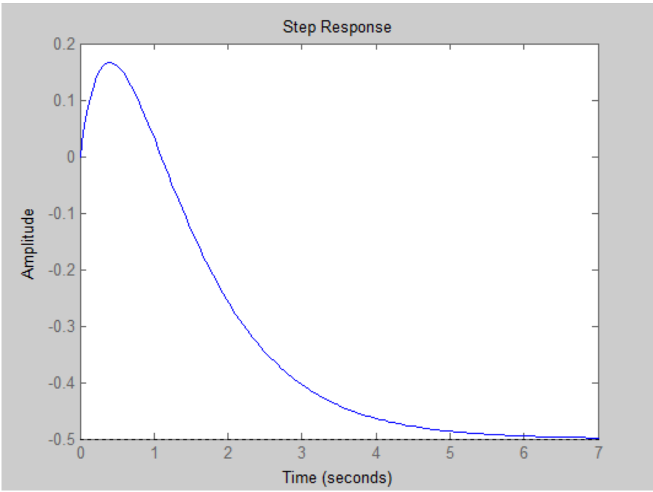

Figure 3

Problem 7

$G_1(s) = \dfrac{4}{s+4}, G_2(s) = \dfrac{2}{s+2}$

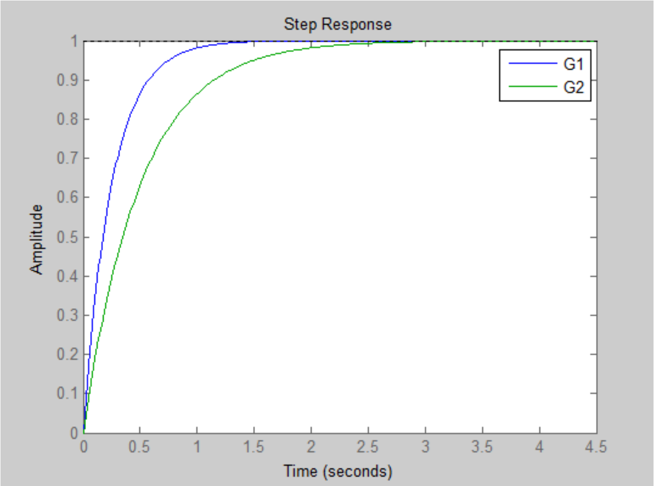

Figure 4

Problem 8

$$g(\infty) = \lim_{s\to 0} sG(s) = \lim_{s\to 0} \frac{20}{s^2+5s+k}$$
$$= \frac{20}{k} = 10$$
$$k = \frac{20}{10} = 2$$

Problem 9

$$s^2 + 10s + 75 = 0$$
$$s = -5 \pm 5\sqrt{2}j$$

Response of the system is under-damped.

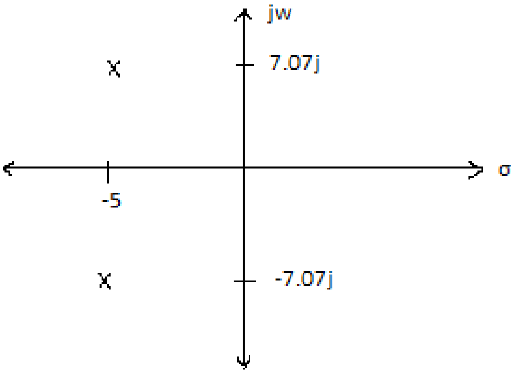

Figure 5

Problem 10

First consider the second order system only

$$G(s) = \frac{150}{s^2+10s+75}$$

For this second order system settling time is:

$$t_s = \frac{4}{\sigma_d} = \frac{4}{5} = 0.8sec$$

According to second order proximity criteria we have to place third pole as:

$$p > 5(\sigma_d) = 5(5) = 25$$

Let $p = 30$

Problem 11

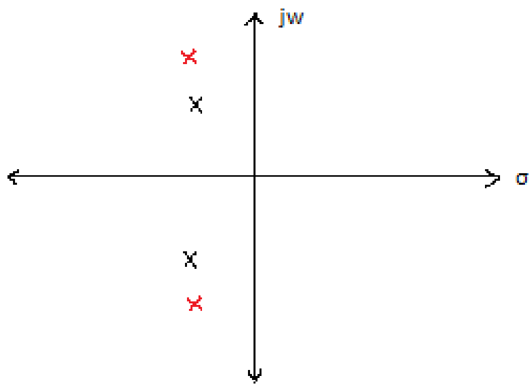

Figure 6

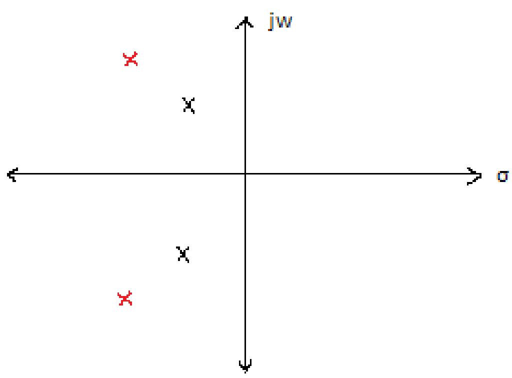

Figure 7

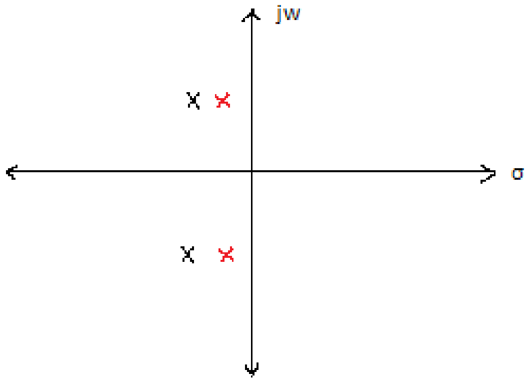

Figure 8

Problem 12

$$r(t) = \cos(2t)$$

Problem 13

Applying Routh-Hurwitz criterion to the denominator of transfer function. The routh table is given by

s^3	1	4	0
s^2	3	k+1	0
s^1	$\frac{12-(k+1)}{3}$	0	0
s^0	20	0	0

For the system to be BIBO stable all poles should be in left half plane it means there must be no sign change in first column of the table so:

$$\frac{12-(k+1)}{3} > 0$$
$$k+1 < 12$$

$$k < 11 \tag{13.1}$$

$$k+1 > 0$$

$$k < -1 \tag{13.2}$$

From Eq(13.1) and Eq(13.2)

$$-1 < k < 11$$

Problem 14

$$\frac{Y(s)}{R(s)} = \frac{KG(s)}{1+KG(s)}$$

$$= \frac{\dfrac{K}{s^3+3s^2+4s+1}}{1+\dfrac{K}{s^3+3s^2+4s+1}}$$

$$= \frac{K}{s^3 + 3s^2 + 4s + K + 1}$$

$$E(s) = (1 - Y(s))R(s)$$

$$= \left(\frac{s^3 + 3s^2 + 4s + 1}{s^3 + 3s^2 + 4s + K + 1} \right) \frac{1}{s}$$

$$e(\infty) = \lim_{s \to 0} sE(s) = \frac{1}{K+1} < 0.1$$

$$K + 1 > 10$$

$$K > 9$$

But for stability $K < 11$ so:

$$9 < K < 11$$

Problem 15

$$\tau \approx 0.7$$

$$\sigma \approx \frac{1}{0.7} \approx 1.4$$

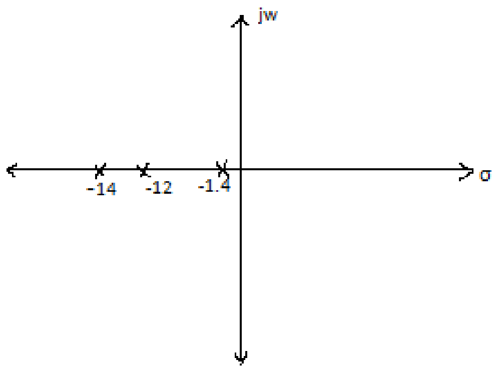

Figure 9