EE361: Control Systems

Midterm Exam (Spring 2019)

Instructor: Usama Bin Sikandar **March 28, 2019**

Student Name: ______________________________

Roll Number: ______________________________

100 Points

120 minutes

Instructions:

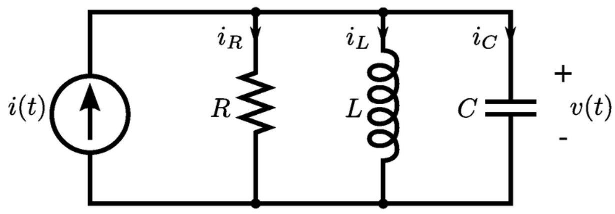
- There are **17** printed pages and **3** blank page.
- **All problems** are compulsory.
- **Calculators** are allowed, except graphical calculators.
- Write **all your work in this booklet**, including any rough work.
- Read the statement **carefully** before you start attempting a problem.
- Properly **label all the axes and relevant points** if you draw any graphs.
- A **formula sheet** is provided at the end of this booklet.

Blank page for marks and contestation.
Do NOT write anything on this page.

P1	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**	**P10**	**P11**	**P12**	**P13**	**P14**	**P15**	**Total**
5	**10**	**5**	**5**	**5**	**10**	**5**	**5**	**5**	**5**	**10**	**5**	**10**	**10**	**5**	**100**

Problem 1 [5 Marks]

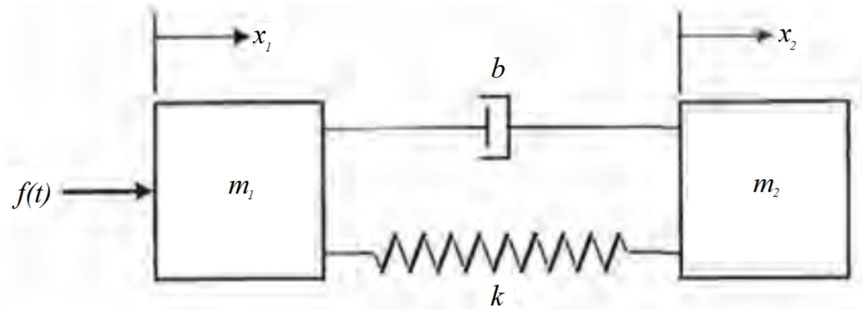
Consider the RLC circuit shown in the figure. The circuit is driven through a current source $i(t)$ and the output voltage $v(t)$ is measured across the capacitor.



Find the transfer function $\dfrac{V(s)}{I(s)}$. [Hint: Use admittance model]

Problem 2 [10 Marks]

A robotic arm with significant flexibility can be modeled as a two-mass system shown in the figure. A heavy load on the gripper can be modeled as a force $f(t)$. Ignore the effect of gravity and any friction.



Derive the transfer function $\dfrac{X_2(s)}{F(s)}$.

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Problem 3 [5 Marks]

Linearize the following nonlinear differential equation about $y = 0$, and find the transfer function between the output y and an appropriate input.

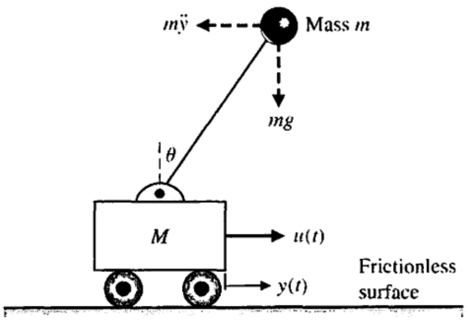
$$\frac{dy}{dt} + 10e^{-y} = 2x$$

Problem 4 [5 Marks]

The problem of balancing an inverted pendulum on a moving cart is shown in the figure. The differential equations governing the system are given below.

$$M\ddot{y} + mg\theta = u,$$
$$Ml\ddot{\theta} - Mg\theta = -u,$$

where u is the input force and l is the length of the pendulum. Find a state-space model for the system if the outputs are θ and $\dot{\theta}$. You are required to use a set of minimum number of state variables for this system.



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Problem 5 [5 Marks]

Find the transfer function corresponding to the following state-space model.

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u$$

$$y = \begin{bmatrix} -1 & 0 \end{bmatrix} x + u$$

Problem 6 [10 Marks]

The transfer function for the position of the vehicle with respect to the engine's driving thrust is given by

$$G(s) = \frac{s-1}{(s+1)(s+2)}.$$

Evaluate the response of the vehicle to a unit step driving thrust and sketch its graph.

Problem 7 [5 Marks]

For each of the following transfer functions, sketch the step response on the same graph. Label the responses clearly as y_1 and y_2, marking all the important points on the axes.

[Hint: You do not need to evaluate the step response functions to sketch these graphs.]

$$G_1(s) = \frac{4}{s+4}, \quad G_2(s) = \frac{2}{s+2}$$

Problem 8 [5 Marks]

Find the value of k for which the unit impulse response of the following transfer function approaches 10 in steady state.

$$G(s) = \frac{20}{s(s^2 + 5s + k)}$$

Problem 9 [5 Marks]

Plot the poles of the following transfer function in the complex plane. What type of damping is there in the system (underdamped, overdamped, critically damped or undamped)?

$$G(s) = \frac{150}{s^2 + 10s + 75}$$

Problem 10 [5 Marks]

Find a value of p for which the step response of the following transfer function has a settling time of approximately 0.8 s.

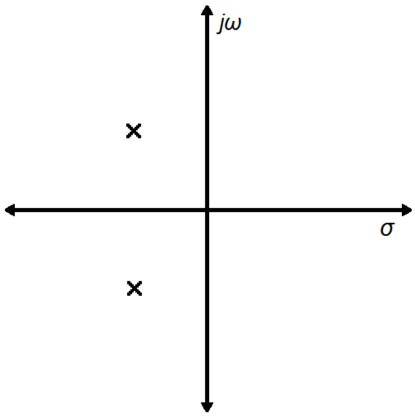
[Hint: First find the settling time of a relevant 2nd order transfer function.]

$$G(s) = \frac{150}{(s+p)(s^2+10s+75)}$$

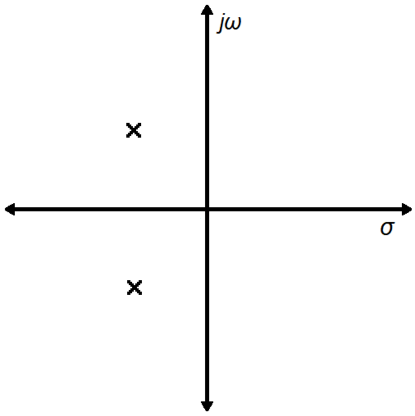
Problem 11 [10 Marks]

Each of the following figures shows the location of poles of a second order system. Draw the new location of the poles to meet the given specification for the step response. Explain very briefly in each case.

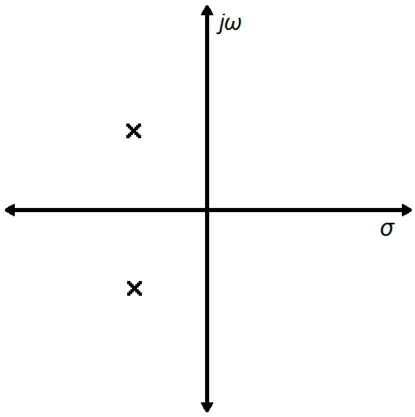
(a) Peak time decreases but settling time is unchanged.



(b) Settling time decreases but overshoot is unchanged.



(c) Overshoot increases but peak time is unchanged.



Problem 12 [5 Marks]

For the following transfer function, give an example of a bounded input function $r(t)$ that would cause the output $y(t)$ to increase without bounds i.e. output is unbounded. [Hint: Resonance]

$$G(s) = \frac{4}{(s+1)^2(s^2+4)}$$

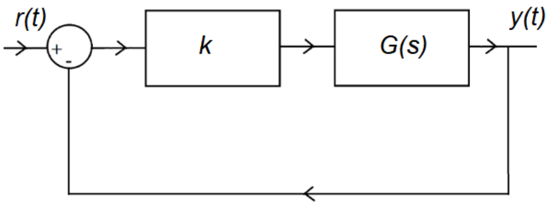
Problem 13 [10 Marks]

For the following transfer function, find the range of values of k for which the corresponding system is BIBO stable.

$$G(s) = \frac{k(s+1)}{s^3 + 3s^2 + 4s + k + 1}$$

Problem 14 [10 Marks]

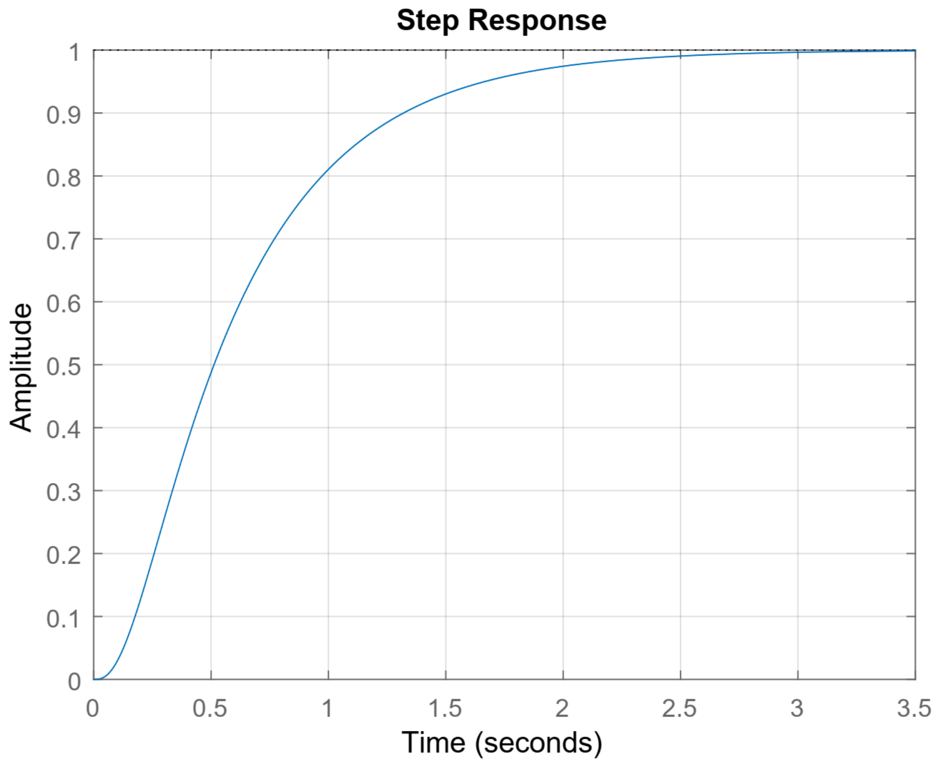
For the following closed-loop system, find the range of values of k for which the steady-state error for a constant input is less than 10%. (It would be a better idea to solve Problem 13 first.)



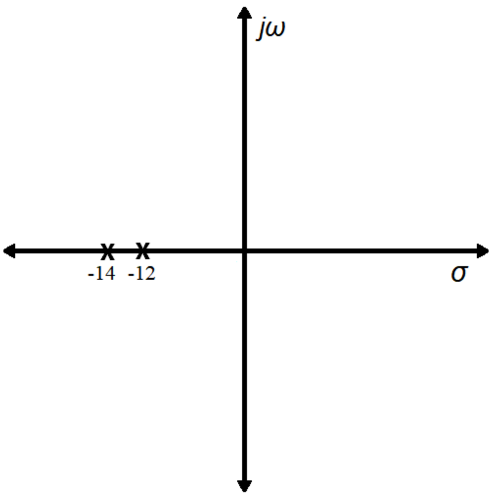
$$G(s) = \frac{1}{s^3 + 3s^2 + 4s + 1}$$

Problem 15 [5 Marks]

The step response of a third order transfer function with an unknown third pole is shown in the figure below.



Two poles of this transfer function are shown below in the complex plane.



Using the information in the step response, guess the approximate location of the third pole and mark its location and value on the complex plane. Explain your reasoning clearly otherwise no credit will be given. [Hint: Time constant]

Taylor Series

$$f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(x_0)}{k!}(x - x_0)^k$$

Damping

$$\begin{array}{rc} \text{Undamped} & \zeta = 0 \\ \text{Underdamped} & 0 < \zeta < 1 \\ \text{Critically damped} & \zeta = 1 \\ \text{Overdamped} & \zeta > 1 \end{array}$$

Response of Second Order Underdamped Systems

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}}$$

$$t_s = \frac{4}{\sigma_d} = \frac{4}{\zeta\omega_n}$$

$$t_r = \frac{\pi - \theta}{\omega_n}$$

$$\%O.S. = 100 e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}$$

$$O.S. = 5\%, \quad \zeta = 0.7$$

$$O.S. = 15\%, \quad \zeta = 0.5$$

$$O.S. = 35\%, \quad \zeta = 0.3$$

Laplace Transform

$$F(s) = \int_0^{\infty} f(t) e^{-st} dt$$

Laplace Transforms Table

$$\begin{array}{rcl} \delta(t) & \longmapsto & 1 \\ h(t) & \longmapsto & \frac{1}{s} \\ e^{at} & \longmapsto & \dfrac{1}{s-a} \\ e^{at} f(t) & \longmapsto & F(s-a) \\ \cos \omega t & \longmapsto & \dfrac{s}{s^2 + \omega^2} \\ \sin \omega t & \longmapsto & \dfrac{\omega}{s^2 + \omega^2} \\ t^n & \longmapsto & \dfrac{n!}{s^{n+1}} \\ f'(t) & \longmapsto & sF(s) - f(0) \\ f''(t) & \longmapsto & s^2 F(s) - sf(0) - f'(0) \end{array}$$