Problem 1
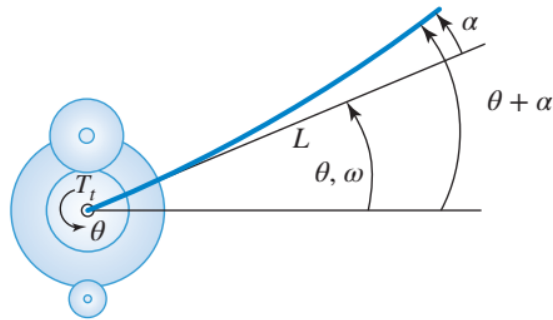
The study of the flexible links, such as the one shown in the following figure, is important because of their application to the control of flexible lightweight robots.

The flexible link angle is deflected by a servomotor. Given that only the base angle, θ, can be measured but the tip angular deflection relative to the undeformed link, α, is very hard to measure. For a specific setup, a state-space model of the system was developed. The state vector is $\mathbf{x} = \begin{bmatrix} \theta & \alpha & \omega & \dot{\alpha} \end{bmatrix}^{\top}$, where $\omega = \dot{\theta}$ and input u is the voltage applied to the servomotor. Thus the system is represented as

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u, \qquad y = \mathbf{C}\mathbf{x}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 673.07 & -35.1667 & 0 \\ 0 & -1023.07 & 35.1667 & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 61.7325 \\ -61.7325 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}.$$

(a) Draw a signal-flow graph of the system and, from this graph, determine whether the system is controllable and/or observable. Explain your answer clearly.

Signal-flow graph of the system is given below in the figure.It is clear from the figure that the system is controllable and observable. We can control all the states with the input u and also can estimate all states through output x_1.
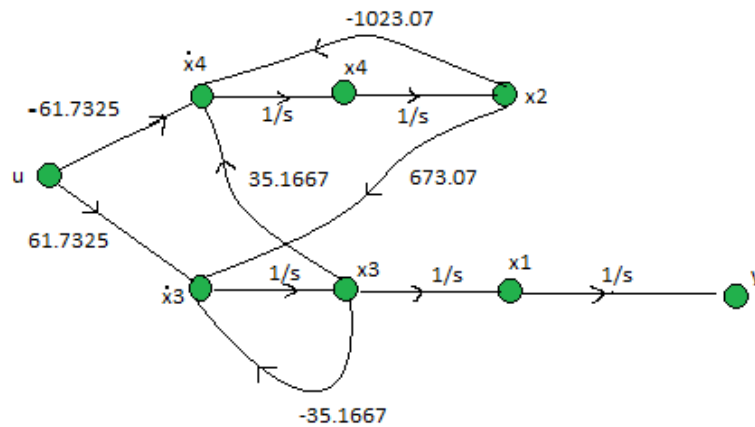
(b) Evaluate the system's controllability matrix using MATLAB. With this matrix, determine whether the system is controllable or not.

Through MATLAB we can evaluate that the determinant of the controllability matrix is non-zero (-1.7791×10^{12}) so the system is controllable.

```
A=[0 0 1 0;0 0 0 1;0 673.07 -35.1667 0;0 -1023.07 35.1667 0];
B=[0 0 61.7325 -61.7325]';
C=[1 0 0 0];
Cm=ctrb(A,B);
D=det(Cm)
```

(c) Evaluate the system's observability matrix using MATLAB. With this matrix, determine whether the system is observable or not.

Through MATLAB we can evaluate that the determinant of the observability matrix is non-zero (-4.5×10^5) so the system is observable.

```
A=[0 0 1 0;0 0 0 1;0 673.07 -35.1667 0;0 -1023.07 35.1667 0];
B=[0 0 61.7325 -61.7325]';
C=[1 0 0 0];
Om=obsv(A,C);
D1=det(Om)
```

(d) It is desired to build a full state-feedback compensation around this system so that the system's characteristic equation becomes $D(s) = (s + 15)^4$. Using MATLAB, find a feedback gain matrix $\mathbf{K}$ that will place the closed-loop poles in the desired location.

Gain matrix that will place the closed-loop poles of the system in desired location is

$$\mathbf{K} = \begin{bmatrix} 2.3431 & -2.9529 & 0.0552 & -0.3471 \end{bmatrix}$$

```
A=[0 0 1 0;0 0 0 1;0 673.07 -35.1667 0;0 -1023.07 35.1667 0];
B=[0 0 61.7325 -61.7325]';
C=[1 0 0 0];
p1=[-15 -15 -15 -15];    % poles of desired polynomail D(s)=(s+15)^4
K=acker(A,B,p1)
```

(e) In order to implement full state-feedback, we need to design an observer to estimate all the states. Place the observer poles at appropriate location and, using MATLAB, find an observer gain matrix **L** that will place the observer poles in the desired location.
In order to make a stable system we have to place the poles in L.H.P.You can select any location in L.H.P to place poles in order to design an observer.As we want estimate of our states as soon as possible so it's a good practice to place poles of the observer at higher values than poles of our feed-back gain.Gain matrix that will place the observer poles in desired location $(-30, -35, -40, -45)$ is

$$\mathbf{L} = \begin{bmatrix} 114.833 \\ 113.5976 \\ 3313.621 \\ -4328.6358 \end{bmatrix}$$

```
A=[0 0 1 0;0 0 0 1;0 673.07 -35.1667 0;0 -1023.07 35.1667 0];
B=[0 0 61.7325 -61.7325]';
C=[1 0 0 0];
p2=[-30 -35 -40 -45]; %desired poles
L=place(A',C',p2)
```

(f) Simulate the system in Simulink, with your controller and observer, and plot a graph of the system's unit step response. Attach a print-out of this graph with your homework.
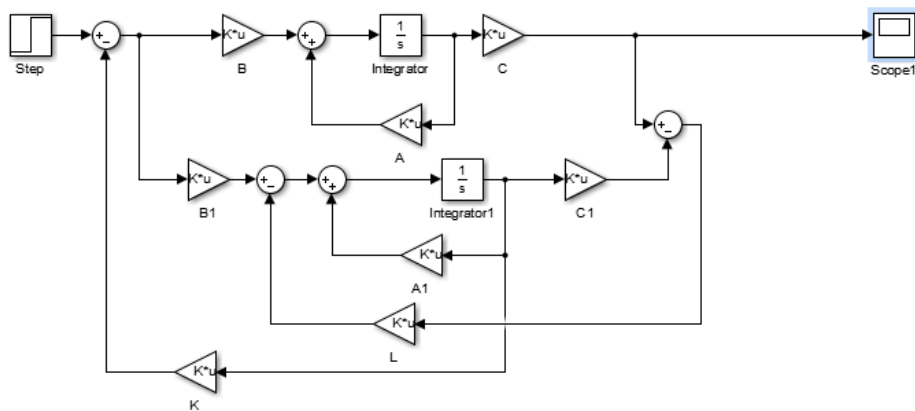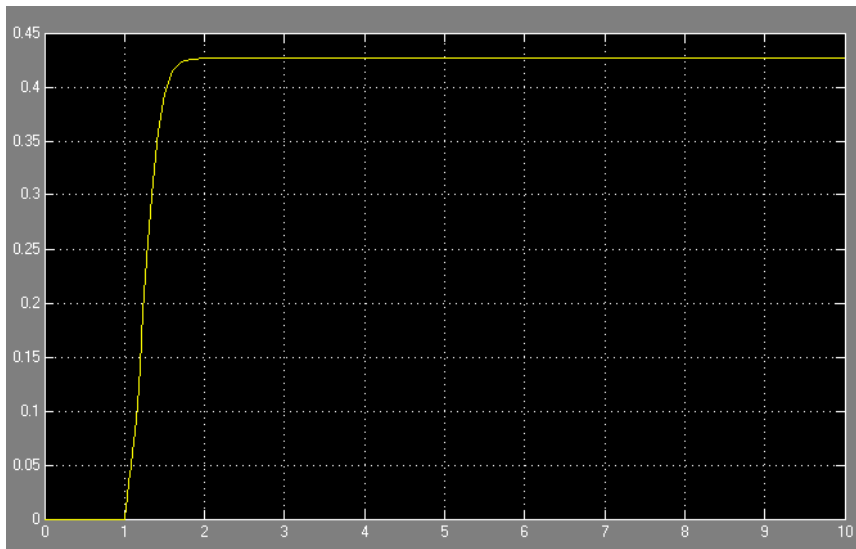

Figure 0.1: simulink model
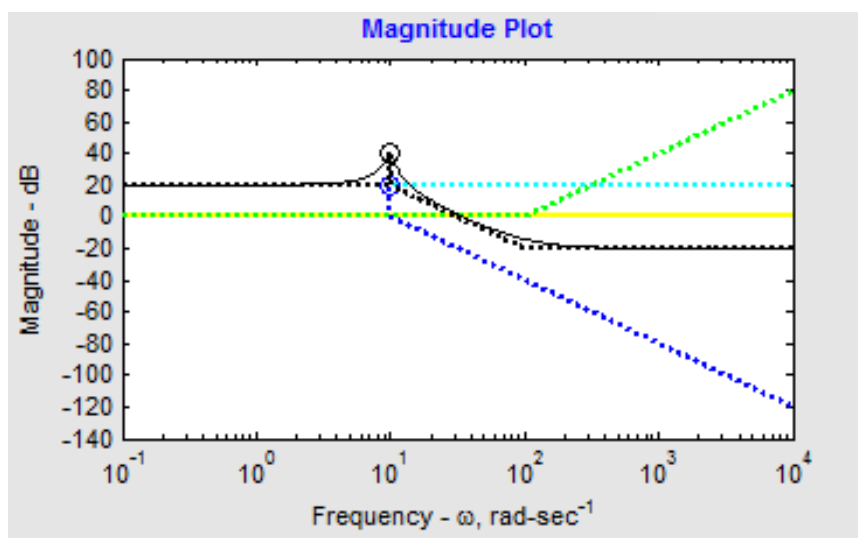
Figure 0.2: Step response

Problem 2

Sketch Bode plot of the following transfer function using rules of sketching Bode plots. No credit will be given if the sketch is drawn with the help of MATLAB.
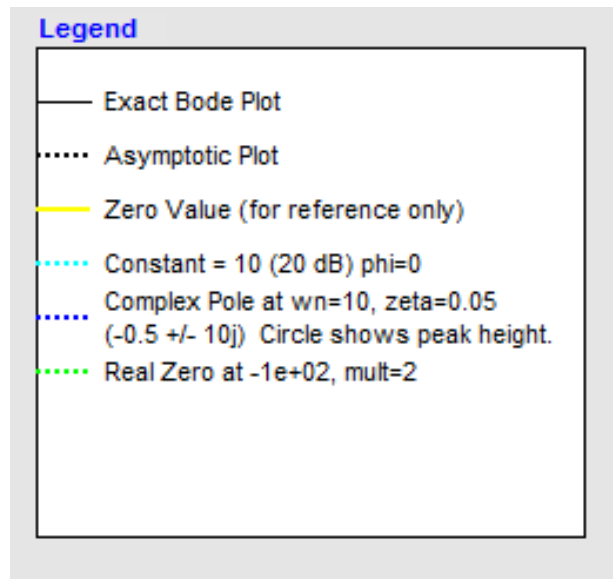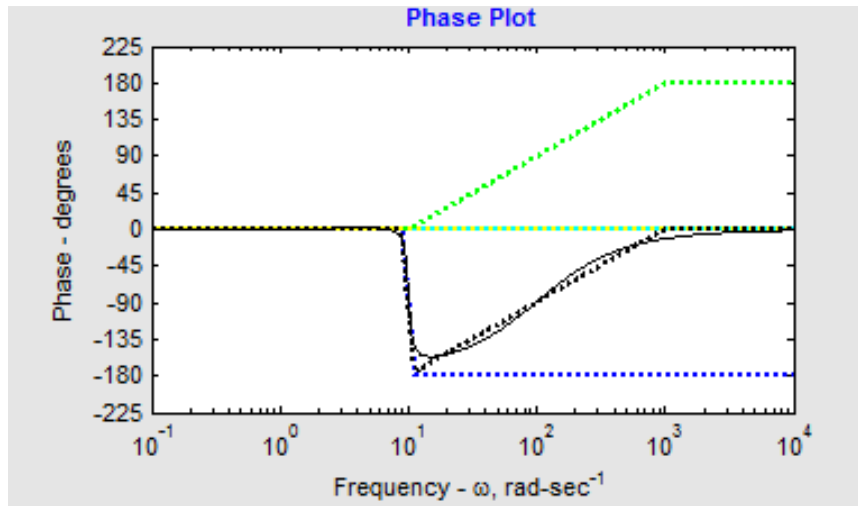
$$G(s) = \frac{0.1(s + 100)^2}{s^2 + s + 100}$$

Rearrange the transfer function to get the standard form where $\omega_n = 10, \zeta = 0.05$

$$G(s) = \frac{10 \left(\dfrac{s}{100} + 1 \right) \left(\dfrac{s}{100} + 1 \right)}{\dfrac{s^2}{100} + \dfrac{s}{10} + 1}$$

The transfer function has a double left half-plane zero at $\omega = 100$ and a second order complex conjugate pole at $\omega = 10$ with $\zeta = 0.05$ that will cause a peak of $-20 \log_{10}(2\zeta) \approx 20 dB$.

Phase Plot

Legend

- —— Exact Bode Plot
- ····· Asymptotic Plot
- —— Zero Value (for reference only)
- ····· Constant = 10 (20 dB) phi=0
- ····· Complex Pole at wn=10, zeta=0.05 (-0.5 +/- 10j) Circle shows peak height.
- ····· Real Zero at -1e+02, mult=2

Problem 3

Guess a possible transfer function corresponding to the following Bode plot. Show your working clearly.

Red lines in the figure below are drawn to identify the points where graph is changing.In first region there is a constant line of $20dB$ so take anti-log of this value to get corresponding value in s-plane which is 10.Phase of 180 deg shows presence of two zeros at the same location.We have to identify carefully the location of zeros and increase in magnitude slop indecates that the zeros are at $\omega = 1$There is a change in graph at $\omega = 0.001$ and in second region slop of graph is decreasing with $20dB/decade$ which corresponds to existence of a pole at this location so we have one pole at

$$\frac{s}{0.001} + 1$$
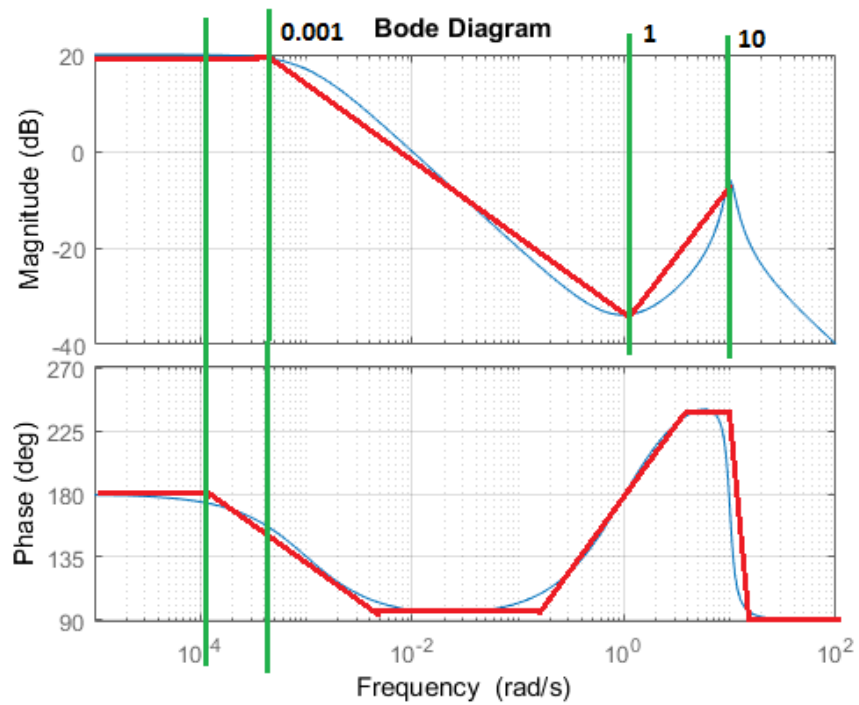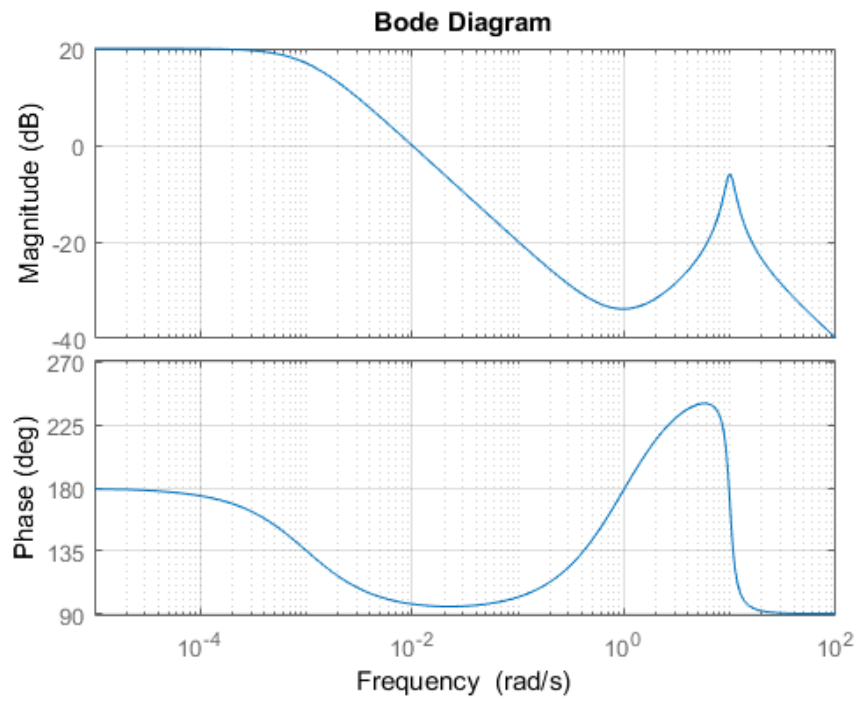
In next region again there is a change in slop at $\omega = 10dB$ is due to the appearance of complex conjugate poles and we can calculate the value of ζ from the peak value which is $20\log_{10}(2\zeta)$ so $\zeta = 0.1$

$$\frac{1}{\dfrac{s^2}{100} + \dfrac{0.2s}{10} + 1}$$

By combining and rearranging all these we get the transfer function

$$G(s) = \frac{-10s^2 - 20s - 10}{10s^3 + 20.01s^2 + 1000s + 1}$$

Put this transfer function in MATLAB to verify the answer