EE361: Control Systems

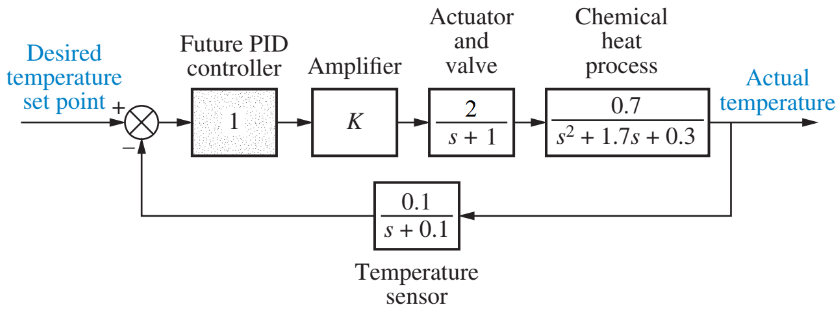
Homework 4 Solution

Due Mon Apr 15, 1:45 PM

Spring 2019

Problem 1

A common application of control systems is in regulating the temperature of a chemical process. The flow of a chemical reactant to a process is controlled by an actuator and valve. The reactant causes the temperature in the vat to change. This temperature is sensed and compared to a desired set-point temperature in a closed loop, where the flow of reactant is adjusted to yield the desired temperature. In class, we will soon learn how a PID controller is used to improve the performance of such process control systems. Figure shows the control system before the addition of the PID controller. The PID controller is replaced by the shaded box with a unity gain.



(a) Convert the system into an equivalent unity feedback configuration and find the open-loop transfer function $T_{\mathrm{OL}}(s)$ of the unity feedback system.
We know that:

$$G(s) = (G_1(s))(G_2(s))(G_3(s))(G_4(s))$$

$$G(s) = (1)(K)\left(\frac{2}{s+1}\right)\left(\frac{0.7}{s^2+1.7s+0.3}\right)$$

$$G(s) = \frac{1.4K}{s^3+2.7s^2+2s+0.3}$$

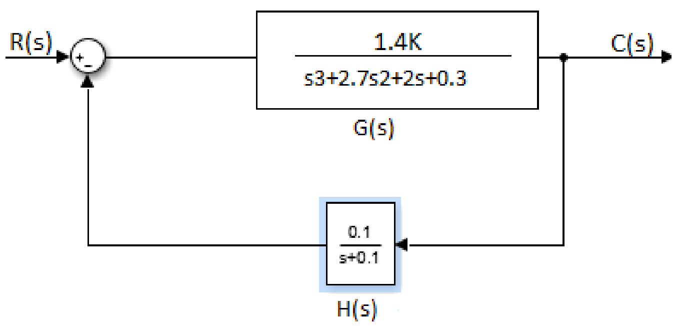


Figure 1: Feedback control

$$G(s) = \frac{1.4K}{(s+1)(s^2+1.7s+0.3)}$$

The close loop system with unity negative feedback will be:

$$T_{oL}(s) = \frac{G(s)}{1 + G(s)(H(s) - 1)}$$

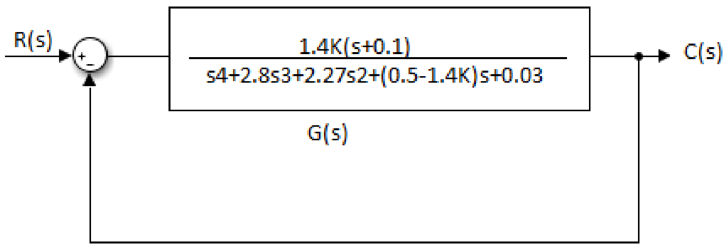


Figure 2: Unity negative feedback control

$$T_{oL}(s) = \frac{\dfrac{1.4K}{(s+1)(s^2+1.7s+0.3)}}{1 + \left(\dfrac{1.4K}{(s+1)(s^2+1.7s+0.3)}\right)\left(\dfrac{0.1}{s+0.1} - 1\right)}$$

$$T_{oL}(s) = \frac{\dfrac{1.4K}{(s+1)(s^2+1.7s+0.3)}}{1 - \left(\dfrac{1.4K}{(s+1)(s^2+1.7s+0.3)}\right)\left(\dfrac{s}{s+0.1}\right)}$$

$$T_{oL}(s) = \frac{1.4K(s+0.1)}{s^4 + 2.8s^3 + 2.27s^2 + (0.5 - 1.4K)s + 0.03}$$

(b) What is the system type? Give a reason for your answer.
No pole is at origin in T_{oL}.
Hence, it is type "0" system.

(c) In terms of K, find the steady-state error in the response of this system for a unit step input.
The steady state error of the system $e_{ss}(\infty)$ for unit step input is:

$$e_{ss} = \lim_{s\to 0} \frac{1}{1 + T_{oL}(s)}$$

$$\lim_{s\to 0}[T_{oL}(s)] = \frac{1.4K}{0.03} = \frac{14K}{3}$$

$$e_{ss} = \frac{1}{1 + \dfrac{14K}{3}}$$

(d) In terms of K, find the steady-state error in the response of this system for a unit ramp input.
The steady state error of the system $e_{ss}(\infty)$ for unit ramp input is:

$$e_{ss} = \lim_{s\to 0} \frac{1}{sT_{oL}(s)}$$

$$\lim_{s\to 0}[sT_{oL}(s)] = 0$$
$$e_{ss} = \infty$$

(e) Find the closed-loop transfer function $T(s)$ of this system.
Closed loop transfer function is:

$$T(s) = \frac{T_{oL}(s)}{1 + T_{oL}(s)}$$

$$T(s) = \frac{\frac{1.4K(s+0.1)}{s^4 + 2.8s^3 + 2.27s^2 + (0.5 - 1.4K)s + 0.03}}{1 + \frac{1.4K(s+0.1)}{s^4 + 2.8s^3 + 2.27s^2 + (0.5 - 1.4K)s + 0.03}}$$

$$T(s) = \frac{1.4K(s+0.1)}{s^4 + 2.8s^3 + 2.27s^2 + 0.5s + (0.03 + 0.14K)}$$

(f) For this system, find the range of amplifier gain, K, to keep the system stable. (Hint: Routh-Hurwitz).
Routh Table for the given system is:

s^4	1	2.27	0.03+0.14K
s^3	2.8	0.5	0
s^2	2.0914	0.03+0.14K	0
s^1	0.4598-0.1874 K	0	0
s^0	0.03+0.14K	0	0

For the system to be stable, there must be no sign changes in Routh array,so

$$0.4598 - 0.1874K > 0$$
$$K < 2.45$$

and

$$0.03 + 0.14K > 0$$
$$K > -0.214$$

Range of amplifier gain K to keep the system stable is

$$-0.214 < K < 2.45$$

(g) Looking at the transfer functions in the figure and $T(s)$ that you found, what do you think is the effect of feedback on the stability of the system? With feedback is it more stable or less stable?

$$G(s) = \frac{1.4K}{(s+1)(s^2 + 1.7s + 0.3)}$$

poles of the open loop transfer function are $s_1 = -1$,$s_2 = -1.5$,$s_3 = -0.2$
All poles of open loop transfer function are in left half plane, that means system is stable without feedback.

$$T(s) = \frac{1.4K(s+0.1)}{s^4 + 2.8s^3 + 2.27s^2 + 0.5s + (0.03 + 0.14K)}$$

For some value of $K(K > 2.45$ and $K < -0.214)$ closed loop system might be unstable.So, the feedback can make the system unstable.

(h) Find the range of K to keep the steady-state error less than 25% for a step input. (Note: Steady-state error analysis only works for stable systems. So the range of K must fulfill the stability condition as well.)

Range of amplifier gain K to keep the system stable is

$$-0.214 < K < 2.45$$

Steady state error should be less than 25%

$$e_{ss} = \frac{1}{1 + \frac{14K}{3}} < 0.25$$

$$3 < 3.5K + 0.75$$

$$K > 0.6485$$

By combining both ranges:

$$0.6485 < K < 2.45$$

(i) Is it possible to keep the steady-state error less than 8% for any value of K? Explain.

$$e_{ss} = \frac{1}{1 + \frac{14K}{3}} < 0.08$$

$$3 < 1.12K + 0.24$$

$$K > 2.46$$

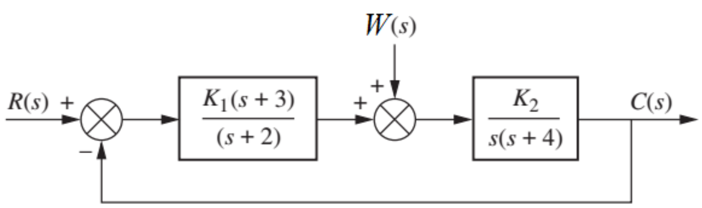
Range of K is :

$$0.6485 < K < 2.46$$

Not possible to to get $e_{ss} = 8\%$ because system is unstable for this range of K.

Problem 2

For the following system, where $W(s)$ is a disturbance signal affecting the system as shown,



(a) Design the values of K_1 and K_2 to meet both the following specifications

- Steady-state error component due to a unit step disturbance is -0.00012

  Let

$$G_1(s) = \frac{K_1(s+3)}{s+2}$$

$$G_2(s) = \frac{K_2}{s(s+4)}$$

$$C(s) = \frac{G_2(s)}{1 + G_1(s)G_2(s)}W(s) + \frac{G_1(s)G_2(s)}{1 + G_1(s)G_2(s)}R(s)$$

$$E(s) = R(s) - C(s)$$

$$E(s) = \frac{R(s)}{1 + G_1(s)G_2(s)} - \frac{G_2(S)}{1 + G_1(s)G_2(s)} W(s)$$

For unit step disturbance :

$$e(\infty) = \lim_{s \to 0} sE(s) = -0.00012$$

Consider part of $E(s)$ due to $W(s)$ only

$$\lim_{s \to 0} s \left(-\frac{G_2(S)}{1 + G_1(s)G_2(s)} \frac{1}{s} \right) = -0.00012$$

$$-\lim_{s \to 0} \frac{K_2(s+2)}{s(s+2)(s+4) + K_1K_2(s+3)} = -0.00012$$

$$-\frac{2K_2}{3K_1K_2} = -0.00012$$

$$K_1 = 5555.56$$

- Steady-state error component due to a unit ramp input is 0.03

  For ramp input $R(s) = \frac{1}{s^2}$ and

$$e(\infty) = \lim_{s \to 0} sE(s) = 0.03$$

Consider part of $E(s)$ due to $R(s)$ only

$$\lim_{s \to 0} s \left(\frac{R(s)}{1 + G_1(s)G_2(s)} \right) = 0.03$$

$$\lim_{s \to 0} s \left(\frac{1}{s^2(1 + G_1(s)G_2(s))}) \right) = 0.03$$

$$\lim_{s \to 0} s \left(\frac{s(s+2)(s+4)}{s^2 \left(s(s+2)(s+4) + K_1K_2(s+3) \right)} \right) = 0.03$$

$$\frac{8}{3K_1K_2} = 0.03$$

$$K_2 = 0.01599$$

(b) Assuming a unit ramp input and a unit step disturbance, find the sensitivity of the disturbance steady-state error for changes in

(i) K_1

$$e(\infty) = \lim_{s \to 0} sE(s)$$

$$e(\infty) = \lim_{s \to 0} s \left(-\frac{G_2(S)}{1 + G_1(s)G_2(s)} W(s) \right)$$

$$e(\infty) = \lim_{s \to 0} s \left(-\frac{G_2(S)}{1 + G_1(s)G_2(s)} \frac{1}{s} \right)$$

$$e(\infty) = -\lim_{s \to 0} \frac{K_2(s+2)}{s(s+2)(s+4) + K_1K_2(s+3)}$$

$$e(\infty) = -\frac{2K_2}{3K_1K_2}$$

$$e(\infty) = -\frac{2}{3K_1}$$

Sensitivity of a function F(s) for gain K is given as:

$$S_{F,K} = \frac{K}{F(s)} \frac{\partial F(s)}{\partial K}$$

For K_1:

$$S_{F,K_1} = \frac{K_1}{\frac{-2}{3K_1}} \frac{2}{3K_1^2}$$

$$S_{F,K_1} = -1$$

(ii) K_2

For K_2:

$$S_{F,K_2} = \frac{K_2}{\frac{-2}{3K_1}} \times 0$$

$$S_{F,K_2} = 0$$

Because

$$\frac{\partial F(s)}{\partial K_2} = 0$$

This shows that sensitivity of disturbance steady-state error is independent of value of K_2