Homework 6

## Some tips to avoid plagiarism cases:

- Do not copy the solutions of your classmates.
- Your are encouraged to discuss the problems with your classmates in whatever way you like but make sure to REPRODUCE YOUR OWN SOLUTIONS in what you submit for grading.
- Cite all the online sources that you get help from.
- Keep your work in a secure place.


## Problem 1

The study of the flexible links, such as the one shown in the following figure, is important because of their application to the control of flexible lightweight robots.


The flexible link angle is deflected by a servomotor. Given that only the base angle, $\theta$, can be measured but the tip angular deflection relative to the undeformed link, $\alpha$, is very hard to measure. For a specific setup, a state-space model of the system was developed. The state vector is $\mathbf{x}=\left[\begin{array}{lll}\theta & \alpha & \omega \\ \dot{\alpha}\end{array}\right]^{\top}$, where $\omega=\dot{\theta}$ and input $u$ is the voltage applied to the servomotor. Thus the system is represented as

$$
\dot{\mathbf{x}}=\mathbf{A} \mathbf{x}+\mathbf{B u}, \quad y=\mathbf{C} \mathbf{x}
$$

where

$$
\mathbf{A}=\left[\begin{array}{cccc}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & 673.07 & -35.1667 & 0 \\
0 & -1023.07 & 35.1667 & 0
\end{array}\right], \quad \mathbf{B}=\left[\begin{array}{c}
0 \\
0 \\
61.7325 \\
-61.7325
\end{array}\right], \quad \mathbf{C}=\left[\begin{array}{llll}
1 & 0 & 0 & 0
\end{array}\right] .
$$

(a) Draw a signal-flow graph of the system and, from this graph, determine whether the system is controllable and/or observable. Explain your answer clearly.
(b) Evaluate the system's controllability matrix using MATLAB. With this matrix, determine whether the system is controllable or not.
(c) Evaluate the system's observability matrix using MATLAB. With this matrix, determine whether the system is observable or not.
(d) It is desired to build a full state-feedback compensation around this system so that the system's characteristic equation becomes $D(s)=(s+15)^{4}$. Using MATLAB, find a feedback gain matrix $\mathbf{K}$ that will place the closed-loop poles in the desired location.
(e) In order to implement full state-feedback, we need to design an observer to estimate all the states. Place the observer poles at appropriate location and, using MATLAB, find an observer gain matrix $\mathbf{L}$ that will place the observer poles in the desired location.
(f) Simulate the system in Simulink, with your controller and observer, and plot a graph of the system's unit step response. Attach a print-out of this graph with your homework.

## Problem 2

Sketch Bode plot of the following transfer function using rules of sketching Bode plots. No credit will be given if the sketch is drawn with the help of MATLAB.

$$
G(s)=\frac{0.1(s+100)^{2}}{s^{2}+s+100}
$$

## Problem 3

Guess a possible transfer function corresponding to the following Bode plot. Show your working clearly.


