EE361: Control Systems		
	Homework 2	
Due Fri Mar 8, 2 PM		*Spring 2019*

Some tips to avoid a plagiarism case:

- Do not copy the solutions of your classmates.
- Your are encouraged to discuss the problems with your classmates in whatever way you like but make sure to REPRODUCE YOUR OWN SOLUTIONS in what you submit for grading.
- Cite all the online sources that you get help from.
- Keep your work in a secure place.

Problem 1

Consider the mass-spring system shown in the figure below. Deja vu? Yes, this is the same system that you dealt with in Homework 1.

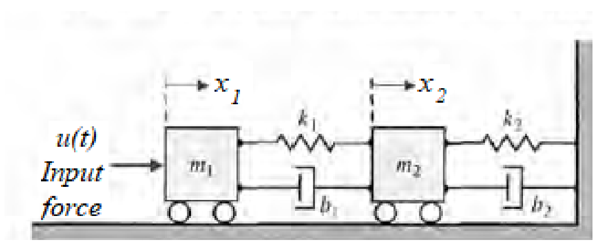

Figure 1: Double mass-spring system

Two carts of masses m_1 and m_2 with negligible rolling friction are connected to two springs (spring constant k_1 and k_2) and dampers (damping constant b_1 and b_2) as shown in the figure. An input force $u(t)$ is applied on m_1 and the output is the position of m_2. The displacements of m_1 and m_2 from their equilibrium positions are x_1 and x_2 respectively.

(a) Write down the differential equation for the displacement x_1 of mass m_1.

(b) Write down the differential equation for displacement x_2 of mass m_2.

(c) Given that the force u is the input and displacement x_2 is the output. Using the state vector $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ p_1 \\ p_2 \end{bmatrix}$ where p_1 and p_2 are momentums of mass 1 and 2 respectively, write down the state-space representation of the system. Clearly specify the matrices $\mathbf{A}, \mathbf{B}, \mathbf{C}$ and $\mathbf{D}$.

(d) Why is the state vector in (c) a valid choice? Explain in terms of energies.

(e) Using the transfer function $\dfrac{X_2(s)}{U(s)}$ that you found in Homework 1, find a different state-space representation of the system. Can you figure out the state vector in this case?

(f) Compare your answers to (c) and (d) and explain the differences, if any.

Problem 2

A space module of mass m descending on Mars is shown in the following figure. The mechanics of its descent can be modeled in the same way we modeled the rocket's equation in class, considering the rate of change of rocket's momentum.

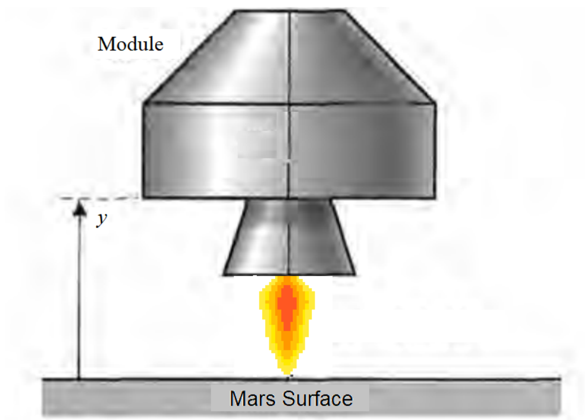

Figure 2: A Space Module Preparing to Land on Mars

(To get a better idea, you can watch a similar landing at `https://www.youtube.com/watch?v=u0-pfzKbh2k`)

The input to this system is the rate at which mass of the fuel burns and is expelled, and the output is the vertical position y of the rocket. Assume that g is the gravity constant on Mars and drag force is proportional to the speed of the module.

(a) What is the order of the system? Explain.

(b) Derive a differential equation for the motion of the space module, in terms of $\dot{y}$, $\ddot{y}$ and $\dot{m}$.

(c) Specify a set of appropriate state variables.

(d) Find a state-space model for this system. Is this a linear model? If not, identify all the nonlinearities in the equations.

[Note: In case your system is nonlinear, you do not need to convert the system in the form of $\mathbf{A}, \mathbf{B}, \mathbf{C}$ and $\mathbf{D}$ matrices.]

Problem 3

Work on your project proposal :)