EE361: Control Systems

Final Exam (Spring 2019)

May 24, 2019

Name: ________________________________ **Roll Number:** ________________

180 Minutes

Instructions

- There are **15** printed pages and **5** blank page.
- **All problems** are compulsory.
- **Calculators** are allowed but you must not borrow from someone else during the exam.
- Write **all your work in this booklet**, including any rough work.
- Read the statement **carefully** before you start attempting a problem.
- Properly **label all the axes and relevant points** if you draw any graphs.
- A **formula sheet** is provided at the end of this booklet.
- This exam will assess your following **Course Learning Objectives** (CLOs)

  CLO 1: Model physical systems using control systems conventions and terminology.
  CLO 2: Analyze a linear system's transient response and stability characteristics.
  CLO 3: Design a suitable controller to track a reference signal and obtain the desired frequency response.

Problem	**1**	**2**	**3**	**4**	**5**	**6**	**Total**
Marks	5	15	20	15	30	15	**100**
	CLO 1		CLO 2		CLO 3		

Course Instructor: Usama Bin Sikandar

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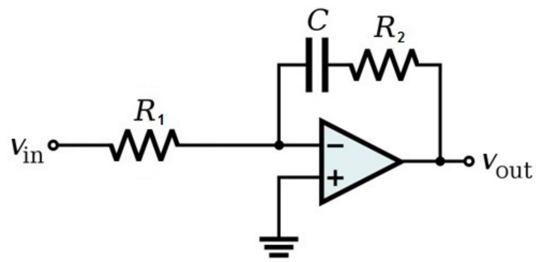
P1	**P2**	**P3**	**P4**	**P5**	**P6**	**Total**
5	**15**	**20**	**15**	**30**	**15**	**100**

Problem 1 [5 Marks]

(a) Find the transfer function of the op-amp circuit shown below.

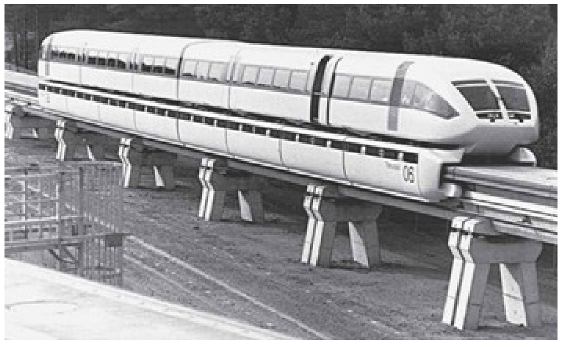


(b) Which one of the following compensators can be implemented through this circuit?

☐ Lead ☐ Lag ☐ Lag-lead ☐ PD ☐ PI ☐ PID

Problem 2 [15 Marks]

A magnetically levitated 'maglev' high-speed train moves on an airgap above the rail system as shown in the figure below.



The transfer function between the size of the airgap and the input current in the electromagnets is as follows.

$$G(s) = \frac{s+4}{s^3 + 14s^2 + 35s - 50}$$

(a) How does moving on an airgap enable the maglev trains to achieve much higher speeds as compared to the trains that move on metal tracks?

(b) Write down a state space representation for this system.

(c) Draw a signal flow graph of the model.

(d) Using your answer to (c), determine whether the system is

☐ Controllable

☐ Observable

Check all that apply. Explain your answer clearly.

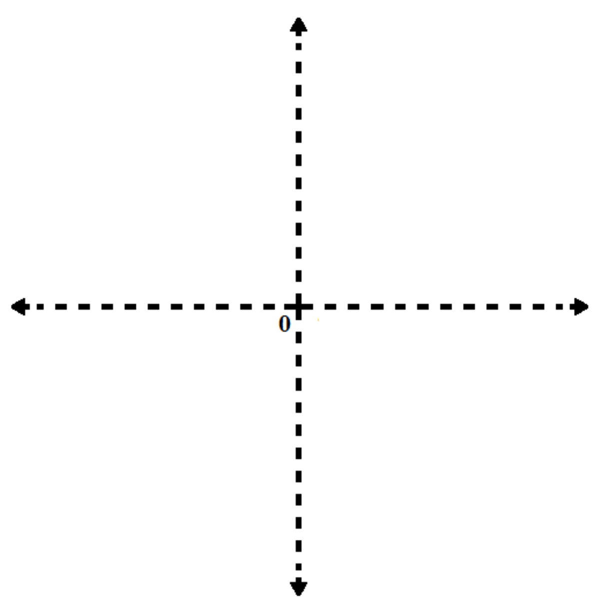
Problem 3 [20 Marks]

Anesthesia induces paralysis (muscle relaxation) and unconsciousness in the patient. Muscle relaxation can be monitored using EMG signals from a nerve in the patient's hand, and unconsciousness can be monitored using cardiovascular system's mean arterial pressure. The anesthesia drug is a mixture of isoflurane and atracurium. An approximate LTI model relating percent induced paralysis $p(t)$ to percent isoflurane $u(t)$ in the mixture is

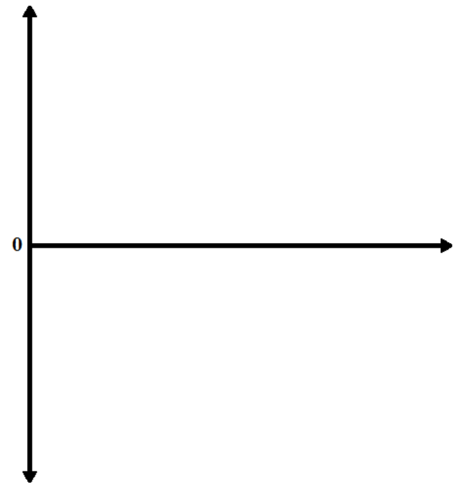
$$\frac{P(s)}{U(s)} = \frac{7.63 \times 10^{-2}}{s^2 + 1.15s + 0.28} \ .$$

(a) Find the natural frequency and damping ratio of the paralysis transient response. What type of damping is there in the system?

(b) Find and plot all the open-loop zeros and poles of the system.



(c) Sketch the graph of paralysis step response if a constant percentage of isoflurane was used in the mixture i.e. sketch the graph of $p(t)$ if the input $u(t) = u_0$ is a constant.



(d) Find the maximum possible percent paralysis if a 2.5% mixture of isoflurane is used. i.e. $u(t) = 2.5$.

(e) Approximately how much time does it take to achieve 98% of the maximum possible paralysis against 2.5% mixture of isoflurane?

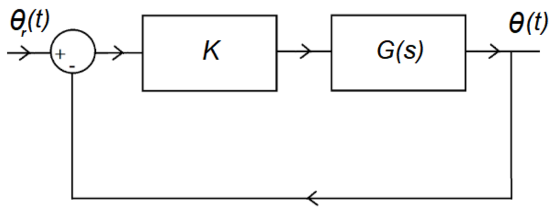
(f) What percentage of isoflurane will induce complete paralysis i.e. $p(\infty) = 100$?

(g) According to the model, is it possible to induce complete paralysis? Explain.

Problem 4 [15 Marks]

Designers have developed small, fast, vertical-take-off fighter aircraft that are invisible to radar (stealth). This aircraft uses quickly turning jet nozzles to steer itself.

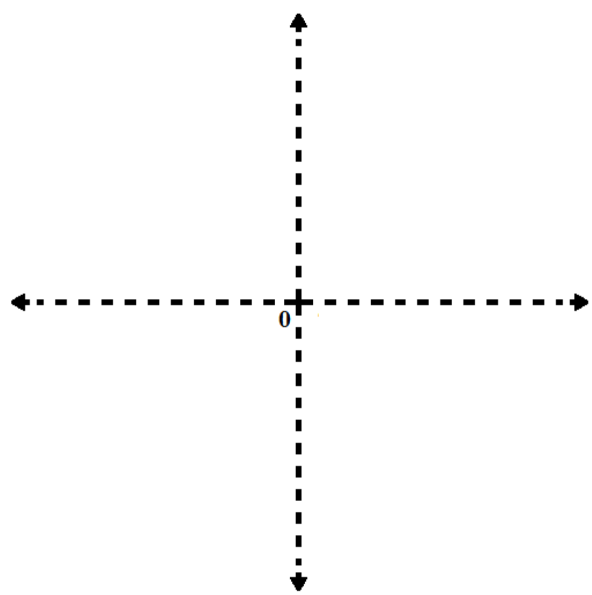
A simple closed-loop system to control the aircraft's heading direction is shown in the figure,



where $G(s) = \dfrac{s+10}{s(s+5)^2}$.

(a) Find the maximum value of gain K for stable operation.

(b) Sketch root locus of the system. Clearly mark the open-loop pole and zero locations, direction of increasing K, and angles and focal point of the asymptotes.

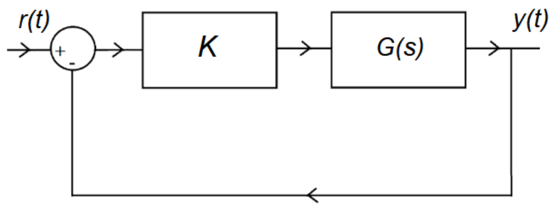


(c) Mark on your root locus the approximate location of closed-loop poles when $K = 200$. Draw a box around each of these poles to distinguish these from open-loop poles.

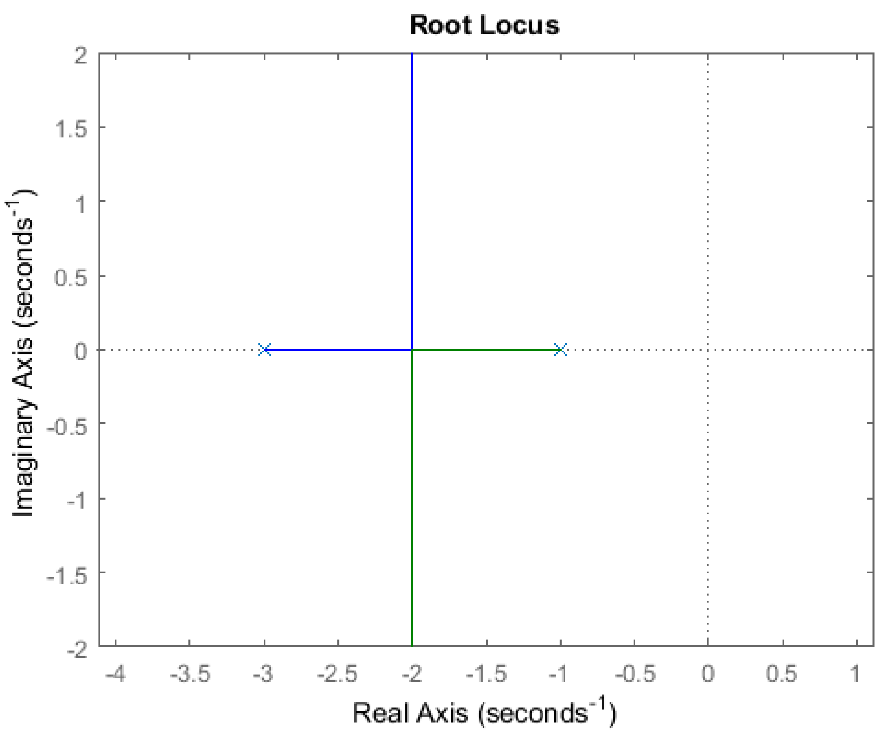
(d) Can this closed-loop system perfectly track a constant angular speed? Explain. (Notice that you are asked about constant angular speed and not constant heading direction.)

Problem 5 [30 Marks]

A system $G(s) = \dfrac{1}{s^2 + 4s + 3}$ with unity feedback and proportional gain K



has the following root locus.



(a) Only using the points on this root locus, estimate the value of gain K that will result in an overshoot of 5% for a unit step input. $\left(\text{Hint: } K = \dfrac{1}{|G(s)|}\right)$

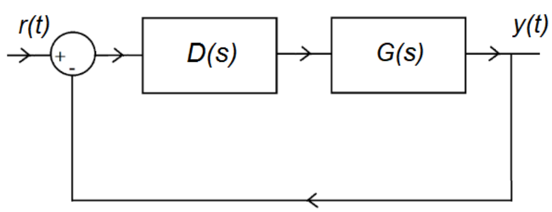
(b) At the operating point given in (a), find the values of settling time, peak time and steady state error.

- $t_s =$
- $t_p =$
- $e_{ss} =$

(c) What are the minimum values of settling time, peak time and steady state error that can be achieved using this simple proportional control? Also give the mimimum value of K to achieve each of these minimum values.

- Minimum $t_s =$ at $K =$
- Minimum $t_p =$ at $K =$
- Minimum $e_{ss} =$ at $K =$

Now let's replace K with a more sophisticated controller $D(s)$ in order to achieve a much wider range of response specifications.



(d) Using root locus techniques, design a PI controller $D(s)$ that achieves a step response with approximately 5% overshoot, 2.7 s settling time and zero steady-state error. Calculate the required values of K_p and K_i.

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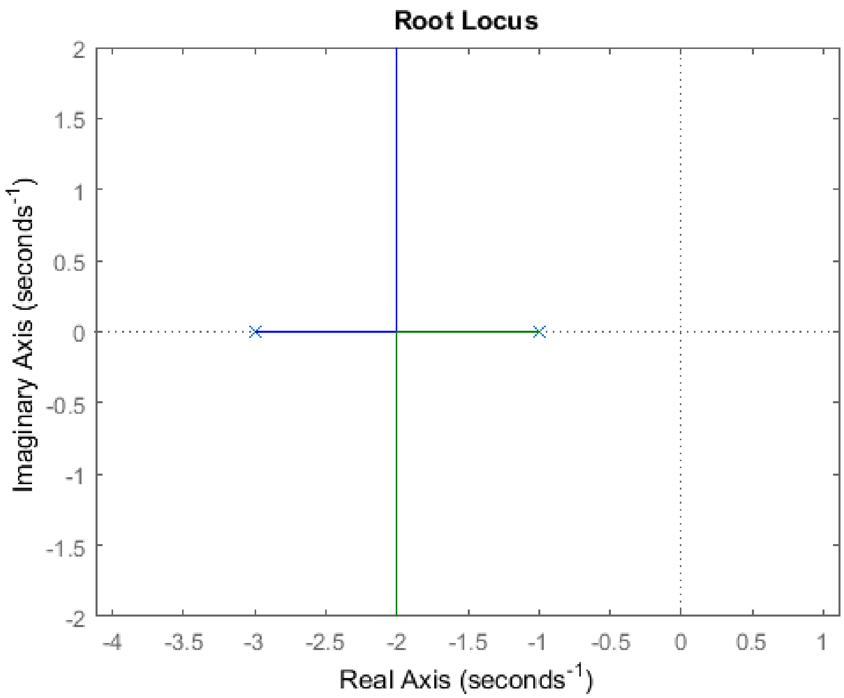
(e) Instead of PI, which passive compensator can be used to achieve the response specifications very close to those given in (d)? Suggest appropriate locations of poles and/or zeros of this compensator and mark them on the following root locus.



(f) Now using pole placement method, design a PID controller $D(s)$ that will yield a step response with approximately 1 s settling time, no overshoot and zero steady-state error.

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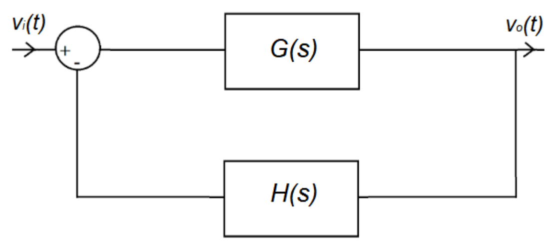
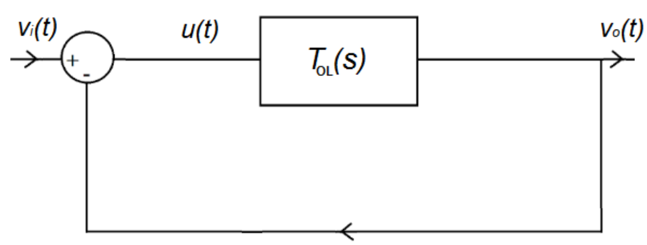
Problem 6 [15 Marks]

Block diagram of an electric circuit is shown in the figure below, where input and output are voltages.

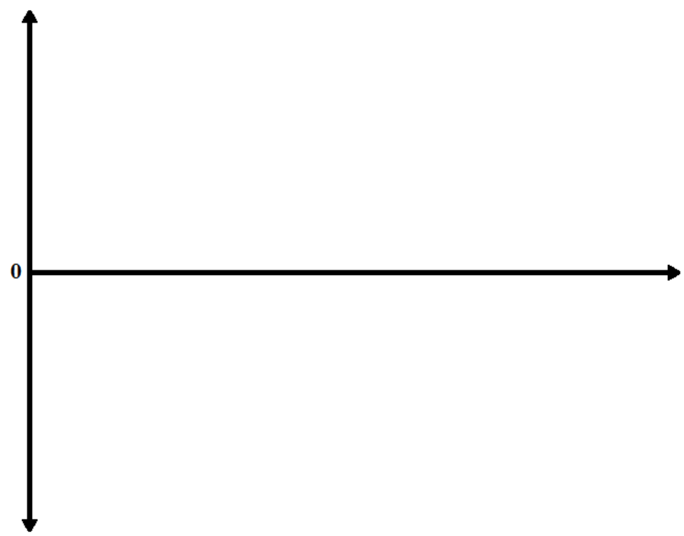


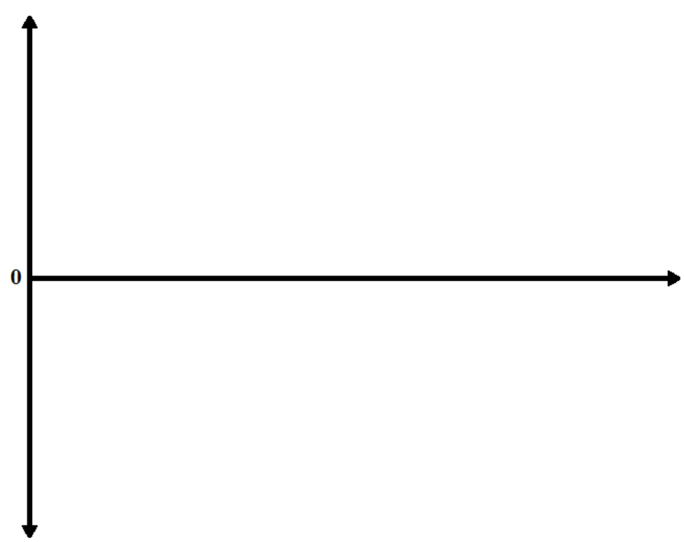
Given that $G(s) = \dfrac{s+1}{s+10}$ and $H(s) = \dfrac{s+10}{s+1}$.

(a) Find $T_{\mathrm{OL}}(s)$ to transform the above system to the following equivalent unity feedback system.



(b) Sketch the Bode plots of $T_{\mathrm{OL}}(s)$. Properly name each axes and mark all the relevant points on the axes.





(c) The steady-state output voltage of the circuit is measured and it turns out to be $v_o(t) = 10\cos 10t$. Using your Bode plots, estimate the input signal $u(t)$ to the block $T_{\mathrm{OL}}(s)$.

(d) Using your answer to (c), estimate the input voltage $v_i(t)$ to the circuit when the steady-state output voltage is $v_o(t) = 10\cos 10t$.

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Taylor Series

$$f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(x_0)}{k!}(x - x_0)^k$$

Dynamic Response of Underdamped Systems

$$t_p = \frac{\pi}{\omega_d}$$
$$t_s = \frac{4}{\sigma_d}$$
$$t_r = \frac{1.8}{\omega_n}$$
$$\%O.S. = 100e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}$$

$$O.S. = 5\%, \quad \zeta = 0.7$$
$$O.S. = 15\%, \quad \zeta = 0.5$$
$$O.S. = 35\%, \quad \zeta = 0.3$$

Root Locus

$$K|G(s)| = 1$$
$$\angle G(s) = 180^\circ$$
$$\sigma_a = \frac{\sum p_i - \sum z_i}{n - m}$$
$$\theta_a = \frac{(2l + 1)180^\circ}{n - m}$$

Laplace Transform

$$F(s) = \int_0^\infty f(t)e^{-st}dt$$

Laplace Transforms Table

$$\delta(t) \longmapsto 1$$
$$h(t) \longmapsto \frac{1}{s}$$
$$e^{at} \longmapsto \frac{1}{s-a}$$
$$e^{at}f(t) \longmapsto F(s-a)$$
$$\cos at \longmapsto \frac{s}{s^2 + a^2}$$
$$\sin at \longmapsto \frac{a}{s^2 + a^2}$$
$$t^n \longmapsto \frac{n!}{s^{n+1}}$$
$$f'(t) \longmapsto sF(s) - f(0)$$
$$f''(t) \longmapsto s^2F(s) - sf(0) - f'(0)$$